MATH 135 Algebra, Assignment 1 Due: Wed Sept 23, 8:30 am

1: (a) Find all real numbers x such that $\dfrac{\sqrt{x}}{1+x} = \dfrac{1}{3-\sqrt{x}}$.

(b) Solve $y = x + \frac{1}{x}$ with $0 < x \leq 1$ for x in terms of y.

2: (a) Find all ordered pairs of integers (x, y) such that $xy = 6 + 2x$.

(b) Find all ordered pairs of integers (x, y) such that $x^2 + y^2 = 4x + 2y$.

3: (a) Determine whether there exists an integer x such that $x^2 + 20$ is a perfect square.

(b) Determine whether there exists an integer x such that $x^2 + 10$ is a perfect square.

4: (a) The first 15 odd prime numbers are $3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47$ and 53. Determine which of these 15 prime numbers are equal to the sum of two squares, then make a conjecture about which odd prime numbers are equal to the sum of two squares.

(b) Find all integers n with $2 \leq n \leq 20$ with the property that n is a factor of $2^{n-1} - 1$, then make a conjecture about which integers $n \geq 2$ have the property that n is a factor of $2^{n-1} - 1$.

5: (a) The first six terms of sequences $\{a_n\}$ and $\{b_n\}$ are listed below:

n	1	2	3	4	5	6
a_n	1	2	5	12	29	70
b_n	1	3	7	17	41	99

(i) Find a rule that governs how the terms a_{n+1} and b_{n+1} are obtained from the terms a_n and b_n, and use this rule to find the terms a_7, b_7, a_8 and b_8.

(ii) Use a calculator to find the ratio ${b_n}^2/{a_n}^2$ for $1 \leq n \leq 6$ and make a conjecture about the limit $\lim\limits_{n\to\infty}(b_n/a_n)$.

(b) For a real number x, let $\lfloor x \rfloor$ denote the largest integer which is less than or equal to x. For each positive integer n, let $a_n = \lfloor n\sqrt{2} \rfloor$ and let b_n be the n^{th} positive integer which does not occur in the sequence $\{a_n\}$. The first 20 terms of the sequence $\{a_n\}$ and the first 8 terms of the sequence $\{b_n\}$ are listed below.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a_n	1	2	4	5	7	8	9	11	12	14	15	16	18	19	21	22	24	25	26	28
b_n	3	6	10	13	17	20	23	27												

Make a conjecture about a rule which allows us to determine the terms a_n and b_n without using a calculator and without using the value of $\sqrt{2}$.