Name (print): _____

Signature: _____

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ECE 206, Advanced Calculus 2

Midterm Test, Spring Term, 2016

University of Waterloo

Instructor: Stephen New

Date: June 17, 2016

Time: 8:30-9:50 am

Instructions:

1. Place your name, signature and ID number in the spaces provided at the top of this page.

2. This test contains 6 pages, including this cover page and a page at the end for rough work.

3. No calculators or any other electronic devices are allowed.

4. Answer all 4 questions; all questions will be given equal value.

5. Provide full explanations with all your solutions.

Question	Mark
1	/10
2	/10
3	/10
4	/10
Total	/40

[10] **1:** (a) Let S be the surface given parametrically by $(x, y, z) = \sigma(s, t) = \left(s - t^2\,,\, \frac{s}{t}\,,\, \sqrt{s\,t}\right)$. Find an implicit equation (in the form $ax + by + cz = d$) for the tangent plane to the surface S at the point where $(s, t) = (4, 1)$.

(b) Let C be the curve of intersection of the paraboloid $z = 1 - x^2 - y^2$ with the plane $z = 1 - 2x$. Find a parametric equation for the tangent line to C at the point $(1, 1, -1)$.

[10] **2:** (a) Let $f(x,y,z) = \dfrac{xy^3}{x+z^2}$, $a = (1,2,1)$ and $u = \frac{1}{3}(2,1,-2)$. Find $D_u f(a)$.

(b) Let $(u,v) = f(x,y) = \Big(xe^{-xy}\,,\, 1 + x^2 + x\sin y\Big)$, let $z = g(u,v) = \sqrt{u^2+2v^2}$, and let $h(x,y) = g\big(f(x,y)\big)$. Find $\nabla h(1,0)$.

(c) Let $F(x,y,z) = \left(z^2 + \dfrac{1}{y}\,,\, z - \dfrac{x}{y^2}\,,\, y + 2(x+1)z\right)$. Find g such that $F = \nabla g$.

[10] **3:** (a) Find the mass of the tetrahedron with vertices at $(0,0,0)$, $(1,0,0)$, $(0,2,0)$ and $(0,2,2)$ with density (mass per unit volume) given by $\rho(x,y,z) = 2z - xy$.

(b) Find the total charge on the cone $S = \left\{(x,y,x)\middle| z = \sqrt{x^2+y^2}\,,\, z \leq 1\right\}$ with charge density (charge per unit area) given by $\rho(x,y,z) = x^2 z$.

[10] **4:** (a) Find the work done by the force $F(x,y,z) = \left(\dfrac{z}{1+x}\,,\,\sqrt{y}\,,\,2x\right)$ acting on a small object moving along the curve C given by $(x,y,z) = \alpha(t) = \left(t\,,\,1+2t^2\,,\,1+t^3\right)$ for $0 \le t \le 2$.

(b) Find the flux of the vector field $F = \left(xy^2\,,\,yz\,,\,x^2z\right)$ outwards across the boundary surface $S = \partial D$ of the cylinder $D = \left\{(x,y,z)\middle| x^2+y^2 \le 1\,,\,0 \le z \le 1\right\}$.

This page is for rough work.