Name (print): _____

Signature: _____

ID Number: _____

ECE 206, Advanced Calculus 2

Midterm Test, Spring Term, 2015

University of Waterloo

Instructor: Stephen New

Date: June 15, 2015

Time: 10:00-11:20 am

Instructions:

1. Place your name, signature and ID number in the spaces provided at the top of this page.

2. This test contains 7 pages, including this cover page and a page at the end for rough work.

3. No calculators or any other electronic devices are allowed.

4. Answer all 5 questions; all questions will be given equal value.

5. Provide full explanations with all your solutions.

Question	Mark
1	/5
2	/5
3	/5
4	/5
5	/5
Total	/25

[5] **1:** Let C be the curve of intersection of the sphere $x^2+y^2+z^2=4x$ with the cone $z=\sqrt{x^2+y^2}$. Let L be the tangent line to the curve C at the point $(1,-1,\sqrt{2})$.

(a) Find a parametric equation for the line L.

(b) Find the point of intersection of the line L with the xy-plane.

[5] **2:** Consider the surface $z = f(x, y) = \dfrac{4}{2 + x^4 + x^2 + y^2}$.

(a) Find the gradient $\nabla f(1, 2)$.

(b) Find the directional derivative $D_u f(1, 2)$ when $u = \left(-\frac{4}{5}, \frac{3}{5}\right)$.

(c) An ant walks along the above surface above the circle $(x - 2)^2 + y^2 = 5$ moving counterclockwise (when looking down from above). Find the value of $\tan\theta$, where θ is the angle (from the horizontal) at which the ant is ascending when it is at the point $\left(1, 2, \frac{1}{2}\right)$.

[5] **3:** Find the total charge in the region $D = \left\{(x,y,z)\middle|\sqrt{\frac{1}{3}(x^2+y^2)} \leq z \leq \sqrt{4-x^2-y^2}\right\}$ where the charge density (charge per unit volume) is given by $f(x,y,z) = x^2$.
(The region D is the intersection of a cone with a sphere).

[5] **4:** Find the work done by the force $F(x, y, z) = (2xz, 1 + z, x^2 + y)$ acting on a small object which moves along the curve given by $(x, y, z) = \alpha(t) = \left(\sqrt{1 - t^2}\,, 1 + t\,, \sqrt{t}\,\right)$ for $0 \leq t \leq 1$ in the following two ways.

(a) Calculate the work W directly from the formula $W = \displaystyle\int_C F \bullet T \; dL$.

(b) Find a scalar potential for F then find the work by finding the change in potential.

[5] **5:** Find the flux of the vector field $F(x, y, z) = (x+y^2, xy+z^2, xy+yz)$ through the boundary surface of the tetrahedron with vertices at $(0, 1, 0)$, $(2, -1, 0)$, $(2, 3, 0)$ and $(0, 1, 2)$.

This page is for rough work. It will not be marked.