Name (print): _____

Signature: _____

ID Number: _____

ECE 206, Advanced Calculus 2

Final Examination, Spring Term, 2015

University of Waterloo

Instructor: Stephen New

Date: August 10, 2015

Time: 9:00-11:30 am

Duration: 2.5 hours

Instructions:

1. Place your name, signature and ID number in the spaces provided at the top of this page.

2. This test contains 8 pages, including this cover page and a page at the end for rough work.

3. You will be provided with 5 pages of formulas.

4. No calculators or any other electronic devices are allowed.

5. Answer all 6 questions; all questions will be given equal value.

6. Provide full explanations with all your solutions.

Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
Total	/60

[10] **1:** Let F be the force field given by $F(x,y,z) = \left(\sqrt{3y}\,,\, \sqrt{6xz}\,,\, y/x\right)$ and let C be the curve given parametrically by $(x,y,z) = \alpha(t) = \left(3t, 3t^2, 2t^3\right)$ for $0 \leq t \leq 1$.

(a) Find the length of the curve C.

(b) Find the work done by the force F acting on an object which moves along C.

[10] **2:** Let $F(x, y, z) = \big(xy^2\,,\, yz^2\,,\, zx^2\big)$.

(a) Determine whether F has a scalar potential.

(b) Find the flux of F outwards through the boundary surface S of the solid cylinder $D = \big\{(x, y, z)\big|x^2 + y^2 \leq 1\,,\, 0 \leq z \leq 1\big\}$.

[10] **3:** A loop of wire follows the square with vertices at $(\pm 1, \pm 1, 0)$ carrying a constant current I in the counterclockwise direction (looking down from above). Find the magnetic field at all points along the z-axis.

[10] **4:** Solve each of the following for $z \in \mathbf{C}$. Express your solutions in Cartesian form.

(a) $z^4 + (2\overline{z})^2 = 0$

(b) $2z = \dfrac{4z - 3}{z + i}$

(c) $2\sin z = e^{iz}$

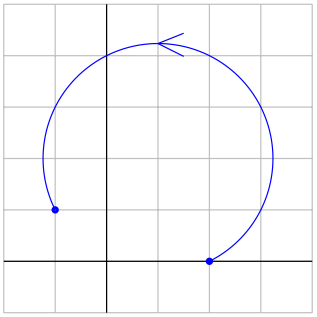
[10] **5:** Let $U = \left\{ r\, e^{i\,\theta} \middle| 0 < r < 1\,,\, 0 < \theta < \frac{\pi}{2} \right\}$.

(a) Find the image of U under the map $w = f(z) = \dfrac{1}{z^2 + 1}$.

(b) Find the temperature $u(z)$ at all points in U given that $u(z)$ is held constant along the boundary with $u(t) = 0$ for $0 \leq t \leq 1$, $u(it) = 0$ for $0 \leq t < 1$ and $u(e^{it}) = 2$ for $0 < t < \frac{\pi}{2}$.

(c) Find the temperature at the point $\frac{1+i}{2}$.

[10] **6:** Find $\displaystyle\int_\alpha \frac{z^2+8}{z^2(z-2i)}\,dz$, where $\alpha(t) = (1+2i) + (1-2i)e^{it}$ for $0 \le t \le \frac{3\pi}{2}$, as shown below.



This page is for rough work. It will not be marked.