Name (print): _____

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ECE 206, Advanced Calculus 2

Final Examination, Spring Term, 2016

University of Waterloo

Instructor: Stephen New

Date: Wednesday August 10, 2016

Time: 9:00 - 11:30 am

Duration: 2.5 hours

Instructions:

1. Place your name, signature and ID number in the spaces provided at the top of this page.

2. This test contains 8 pages, including this cover page and a page at the end for rough work. There will also be 5 pages of formulas.

3. No calculators are allowed.

4. Answer all 6 questions; all questions will be given equal value.

5. Provide full explanations with all your solutions.

Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
Total	/60

[10] **1:** (a) Let $f(x,y) = 2x^2y + \dfrac{y}{x}$. Find every unit vector $u \in \mathbf{R}^2$ such that $D_u f(1,2) = 6$.

(b) Find a parametric equation for the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ with the plane $z = 2y + 1$ at the point $(-1, 2, 5)$.

[10] **2:** Let S be the surface given parametrically by $(x, y, z) = \sigma(u, v) = \big(u^2 + v^2\,,\, u^2 - v^2\,,\, 2uv\big)$.

(a) Find the equation of the tangent plane to the surface S at the point where $(u, v) = (1, 2)$.

(b) Find the area of the portion of the above surface S with $0 \leq u \leq 1$ and $0 \leq v \leq 1$.

[10] **3:** (a) Let $F(x,y,z) = \left(\dfrac{y^2}{z} \, , \, \dfrac{2xy}{z} \, , \, -\dfrac{xy^2}{z^2} \right)$ and let $\alpha(t) = \big(2t^2-1 \, , \, t+1 \, , \, t^2+1\big)$ for $0 \leq t \leq 1$. Find a scalar potential for F then find the work done by F when it acts on a small object which moves along the curve $(x,y,z) = \alpha(t)$.

(b) Find the flux of the vector field $F(x,y,z) = \big(x+yz \, , \, y+xz \, , \, xy+z^2\big)$ outwards across the boundary surface S of the region $D = \big\{(x,y,z) \big| x^2+y^2 \leq z \leq 1\big\}$.

[10] **4:** (a) Use Gauss' Law to find the electric field E at the point $(0, 0, 1)$ which is produced by a static charge distribution of charge density $\rho(x, y, z) = \dfrac{1}{1 + x^2 + y^2 + z^2}$.

(b) Use the Biot-Savard Law to find the x-component of the magnetic field B at the point $(0, 0, 0)$ which is produced by a wire carrying a steady current I along the loop $(x, y, z) = \alpha(t) = \big(\cos^2 t\,,\, \sin t \cos t\,,\, \sin t\big)$ for $0 \le t \le \pi$.

[10] **5:** (a) Solve $(z+1)(z+2i) = iz - 2$ for $z \in \mathbf{C}$. Express the solutions in Cartesian form.

(b) Solve $\tan z = 2i$ for $z \in \mathbf{C}$. Express your solutions in Cartesian form.

(c) Find the area of the image of the line $y = 2x - 5$ under the map $f(z) = \dfrac{10}{z}$.

[10] **6:** (a) Find $\displaystyle\int_\alpha \frac{4}{z^2+4}\,dz$ where α is the line segment $\alpha(t) = i + t\,(2-i)$ for $0 \le t \le 1$.

(b) Find $\displaystyle\int_\alpha \frac{e^{2z}}{(z^2-1)^2}\,dz$ where α is the circle $\alpha(t) = 2\,e^{it}$ for $0 \le t \le 2\pi$.

This page is for rough work.