ECE 206 Advanced Calculus 2, Assignment 9 Due TBA

1: Sketch the image under $f(z) = 10/z$ of the triangle with vertices $2+i$, $4+2i$ and $1+3i$.

2: (a) Evaluate $\tanh\left(\ln 2 + i\,\frac{\pi}{4}\right)$.
(b) Solve $\tanh z = \tanh i\,z$.

3: (a) Show that $\cos^{-1} z = -i\,\log\left(z \pm \sqrt{z^2-1}\right)$, where both sides are multifuctions.
(b) Solve $\cos z = \frac{1}{4}\,(3+i)$.

4: (a) Sketch the image under $f(z) = z^2$ of the circle $z(t) = (2\cos t)\,e^{i\,t}$.
(b) Sketch the image under $f(z) = z^3$ of the line $z(t) = t + i$.

5: Sketch the image under $f(z) = \tanh z$ of the line $z(t) = t + \frac{3\pi}{8}\,i$.

6: (a) For $0 \neq a \in \mathbf{C} = \mathbf{R}^2$ and $0 \neq r \in \mathbf{R}$, show that the circle with diameter a, ta has equation $|z|^2 - (1+t)\,z \cdot a + t\,|a|^2$.
(b) For $0 \neq a \in \mathbf{Z}$ and $0 \neq t \in \mathbf{R}$, show that the image under the map $w = f(z) = \frac{1}{z}$ of the circle with diameter a, ta is the circle with diameter $\frac{1}{ta}, \frac{1}{a}$.