ECE 206 Advanced Calculus 2, Assignment 6 Due TBA

1: A long cylindrical wire of radius R, centred along the z-axis, carries a uniform charge distribution of charge density (charge per unit volume) ρ. Find the electric field E at all points $(x, y, z) \in \mathbf{R}^3$.

2: (a) A circular loop of wire of radius r lies in the xy-plane centred at the origin. The wire carries a constant current I in the counterclockwise direction (looking down from above). Find the magnetic field B at all points on the z-axis.

(b) A circular disc of radius R lies in the xy-plane centred at the origin. The disc carries a uniform charge distribution of charge density (charge per unit area) ρ, and it rotates counterclockwise (looking down from above) at a rate of ω radians per unit time. Find the magnetic field B at the origin.

3: A long thin straight wire lies along the z-axis and carries a constant current I in the positive z direction. The magnetic field surrounding the wire is given by

$$B(x, y, z) = \frac{\mu_0 I}{2\pi}\left(\frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2}, 0\right).$$

(a) Find $\int_\sigma B \cdot N \, dA$ where $(x, y, z) = \sigma(s, t) = \big(s, 0, t\big)$ for $1 \le s \le 3$ and $0 \le t \le 2$.

(b) Find $\int_\alpha B \cdot T \, dL$ where $(x, y, z) = \alpha(t) = \big(4t, t^2 - 1, t^3\big)$ for $-1 \le t \le 2$.

4: The cone given by $z = \sqrt{x^2 + y^2}$ with $x^2 + y^2 \le 4$ carries a nonuniform charge distribution with charge density (charge per unit area) given by $\rho(x, y, z) = z$. Find the electric field E at the point $(0, 0, 2)$.

5: Recall that for a scalar-valued function $f : U \subseteq \mathbf{R}^3 \to \mathbf{R}$ the Laplacian of f is given by $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$. For a vector-valued function $F : U \subseteq \mathbf{R}^3 \to \mathbf{R}^3$, given by $F = (P, Q, R)$, we define the **Laplacian** of F to be

$$\nabla^2 F = \Big(\nabla^2 P, \nabla^2 Q, \nabla^2 R\Big).$$

(a) Show that for $F : U \subseteq \mathbf{R}^3 \to \mathbf{R}^3$ we have $\nabla \times (\nabla \times F) = \nabla(\nabla \cdot F) - \nabla^2 F$.

(b) Show that in a vacuum (where $\rho = 0$ and $J = 0$) the electric and magnetic fields E and B both satisfy the **wave equation**

$$\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \text{and} \quad \nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}.$$

(We remark that it follows from this that $\mu_0 \epsilon_0 = \frac{1}{c^2}$ where c is the speed of light).

6: Find a formula for the gradient of a scalar-valued function in spherical coordinates.