ECE 206 Advanced Calculus 2, Assignment 5 Due TBA

1: Let $F(x,y) = \big(P(x,y), Q(x,y)\big) = (x-y, x+y)$.

(a) Sketch the vector field $\frac{1}{4}F$, along with some of its flow lines.

(b) Verify directly that $\displaystyle\int_C F \bullet T\, dL = \int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$ when $D = \{(x,y)|x^2+y^2 \le 1\}$ and C is the circle given by $(x,y) = (\cos t, \sin t)$.

2: Find the work done by the force $F(x,y,z) = \big(2x+yz\,,\,2y+xz\,,\,2z+xy\big)$ acting on an object which moves along the curve $(x,y,z) = \alpha(t) = \left(\frac{2+t^2}{1+t}\,,\,\frac{2+t^3}{1+t^2}\,,\,\frac{2+t^4}{1+t^3}\right)$ with $0 \le t \le 2$.

3: Find the flux of the vector field $F(x,y,z) = \big(x^2 + \sqrt{yz}\,,\, y + x\sqrt{z}\,,\, z + x\sqrt{y}\big)$ across the boundary surface of the region $D = \big\{(x,y,z)\big|x^2 \le z \le 2-x^2\,,\, 0 \le y \le z\big\}$.

4: Find the work done by the force $F = (y^2z, x+z, x^2+yz)$ acting on an object which moves counterclockwise (when looking down from above) once around the boundary of the surface $S = \big\{(x,y,z)\big|0 \le x \le 1\,,\, 1-x \le y \le 1+x\,,\, z = x^2\big\}$.

5: Find the flux of the vector field $F(x,y,z) = (xz, -yz, 1+y^2)$ across the surface S given by $S = \big\{(x,y,z)\big|z = \cos^{-1}(x^2+y^2)\,,\, x^2+y^2 \le 1\big\}$ with outwards pointing normal vector.

6: Let $F(x,y) = \left(\frac{-y}{x^2+y^2}\,,\,\frac{x}{x^2+y^2}\right)$.

(a) Show that when C is the curve given by $(x,y) = \alpha(t) = \big(r(t)\cos\theta(t)\,,\, r(t)\sin\theta(t)\big)$ for $a \le t \le b$, where $r(t)$ and $\theta(t)$ are smooth with $r(t) > 0$, we have $\displaystyle\int_C F \bullet T\, dL = \theta(b) - \theta(a)$.

(b) Find $\displaystyle\int_C F \bullet T\, dL$ when C is given by $(x,y) = \alpha(t) = (3-t^2, t)$ for $1 \le t \le 2$.