ECE 206 Advanced Calculus 2, Assignment 4 Due TBA

1: A cord, carrying an unevenly distributed charge, is wound around the cone $z = \sqrt{x^2 + y^2}$ following the curve $(x, y, z) = \alpha(t) = (t\cos t, t\sin t, t)$ with $0 \le t \le 4$. The charge density (charge per unit length) of the cord at position (x, y, z) is given by $f(x, y, z) = z$. Find the total charge of the cord.

2: A long evenly charged wire lies along the z-axis. The electric field in the region surrounding the wire is given by $E = 2kq\Big(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}, 0\Big)$, where q is the charge density (charge per unit length) of the wire and k is a constant (which depends on the units used). A small object of unit charge moves along the curve $(x, y, z) = \alpha(t) = \big(1-t, 2t, 1+3t\big)$ for $0 \le t \le 1$. Find the work done by the electric field on the object.

3: A gas expands and rotates with velocity field $V(x, y, z) = (x - y, x + y, z)$. Find the rate (volume per unit time) at which the gas passes through the triangle with vertices at $(1, 0, -1)$, $(1, 3, 2)$ and $(0, 1, 2)$.

4: Let $F(x, y) = (x, 2y)$.

(a) Make an accurate sketch of the vector field $\frac{1}{4}F$.

(b) Find the equations of the flow lines of F.

5: The surface obtained by revolving the circle $(x - 1)^2 + z^2 = 1$ in the xz-plane about the z-axis can be given parametrically by

$$(x, y, z) = \sigma(\theta, \phi) = \Big((1 + \cos\phi)\cos\theta\,,\ (1 + \cos\phi)\sin\theta\,,\ \sin\phi\Big).$$

with $0 \le \theta \le 2\pi$ and $0 \le \phi \le 2\pi$. Find the mass of this surface given that its density (mass per unit area) at position (x, y, z) is given by $f(x, y, z) = 1 + z^2$.

6: Let $F(x, y, z) = (xz, yz, x^2 + y^2)$. Find the flux of F across the boundary surface of the solid given by $x^2 + y^2 \le z \le 1$.