ECE 206 Advanced Calculus 2, Assignment 2 Due TBA

1: Let $(u,v)=f(t)=\big(\cos t+2, 2\sin t-1\big)$ and let $(x,y)=g(u,v)=\left(\frac{u}{v},\frac{v}{u}\right)$. Use the Chain Rule to find the tangent vector to the curve $r(t)=g(f(t))$ at the point where $t=\frac{\pi}{2}$.

2: Let $u=f(x,y,z)=4\,x\tan^{-1}\left(\frac{y}{z}\right)$ where $(x,y,z)=g(s,t)=\left(s^3+t,\sqrt{s}\,t,\frac{t}{s}\right)$. Use the Chain Rule to find $\frac{\partial u}{\partial s}$ and $\frac{\partial u}{\partial t}$ when $(s,t)=(1,-2)$.

3: Let $u=f(x,y,z)=(x+y)e^{y^2+z}$.

(a) Find $\nabla f(1,2,-4)$.

(b) Find the equation of the tangent plane at $(1,2,-4)$ to the surface $f(x,y,z)=3$.

(c) Find $D_uf(1,2,-4)$, where $u=\frac{1}{7}(2,-3,6)$.

4: Let $f(x,y)=x^2y-y^3$. Find $\nabla f(3,-1)$, then for each of the values $m=0,6,6\sqrt{2}$ and 10, find a unit vector u, if one exists, such that $D_uf(3,-1)=m$.

5: A boy is standing at the point $(5,10,2)$ on a hill whose shape is given by

$$z=\frac{600}{100+4x^2+y^2}$$

(where x, y and z are in meters).

(a) At the point where the boy is standing, in which direction is the slope steepest?

(b) If the boy walks southeast, then will he be ascending or descending?

(c) If the boy walks in the direction of steepest slope, then at what angle (from the horizontal) will he be climbing?

6: The temperature around the outer circle of a metal disc of radius 1 meter is held constant, with the top half of the circle held at 0° C and the bottom half of the circle held at 20° C. It can be shown that the temperature at all points of the disc is given by

$$T(x,y)=10+\frac{20}{\pi}\tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right).$$

(a) Sketch the isotherms (level curves of constant temperature) $T=0,5,10,15,20$.

(b) Find $T\left(\frac{1}{2},\frac{1}{2}\right)$ and $\nabla T\left(\frac{1}{2},\frac{1}{2}\right)$.

(c) Find the equation of the tangent line to the isotherm through $\left(\frac{1}{2},\frac{1}{2}\right)$.

(d) Show that if an ant starts at the point $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ (where the temperature is 0°) and it walks on the disc in the direction of ∇T (that is, in the direction in which the temperature increases most rapidly), then it will walk along the circle of radius $\sqrt{3}$ centered at $(2,0)$.