ECE 206 Advanced Calculus 2, Assignment 13 Due TBA

1: Let $f(z) = \dfrac{1}{z^2 - 2z - 3}$.

(a) Find the Taylor series for $f(z)$ centred at 0.

(b) Find the Taylor series for $f(z)$ centred at 1.

(c) Find the radius of convergence of the Taylor series for $f(z)$ centred at $2i$.

2: Let $f(z) = \dfrac{1}{z^2 - 2z - 3}$.

(a) Find the Laurent series for $f(z)$ in $D^*(-1, 4) = \big\{z \in \mathbf{C} \big| 0 < |z + 1| < 4\big\}$.

(b) Find the Laurent series for $f(z)$ in $D^*(3, 4) = \big\{z \in \mathbf{C} \big| 0 < |z - 3| < 4\big\}$.

(c) Find the Laurent series for $f(z)$ in the annulus $A = \big\{z \in \mathbf{C} \big| 2 < |z - 1| < \infty\big\}$.

3: For each of the following functions $f(z)$, find $\displaystyle\int_\alpha f(z)\, dz$ where $\alpha(t) = e^{i\,t}$ for $0 \le t \le 2\pi$.

(a) $f(z) = \dfrac{\coth z}{z^2}$

(b) $f(z) = \dfrac{1 + z}{e^z - (1 + z)}$

(c) $f(z) = z^4 \sin(2/z)$.

4: Find $\displaystyle\int_\alpha \frac{1}{(e^z + 1)^2}\, dz$ where $\alpha(t) = 3i + e^{i\,t}$ for $0 \le t \le 2\pi$.

5: Find $\displaystyle\int_\alpha \frac{1 + z}{1 - \cos z}\, dz$ where $\alpha(t) = 4 + 5\, e^{i\,t}$ for $0 \le t \le 2\pi$.

6: Find $\displaystyle\int_\alpha \frac{\tan z}{z^6}\, dz$ where $\alpha(t) = 1 + 2e^{it}$ for $0 \le t \le 2\pi$.