ECE 206 Advanced Calculus 2, Assignment 12 Due TBA

1: Find $\displaystyle\int_0^{2\pi} \frac{dt}{(5+3\cos t)^2}$.

2: Find $\displaystyle\int_0^{\infty} \frac{x^2\,dx}{x^4+x^2+1}$.

3: Find $\displaystyle\int_0^{\infty} \frac{x\,\sin x}{x^4+4}\,dx$.

4: Find $\displaystyle\int_0^{\infty} \left(\frac{\ln x}{1+x^2}\right)^2 dx$.

5: For a function $f(x)$ with $x \in \mathbf{R}$, the **Fourier transform** of $f(x)$ is defined to be the function $F(\omega) = \mathcal{F}(f)(\omega)$ defined for $s \in \mathbf{R}$ by

$$F(\omega) = \mathcal{F}(f)(\omega) = \int_{-\infty}^{\infty} f(x)\,e^{-i\,\omega x}\,dx\,.$$

Given $F(\omega)$ the function $f(x)$ can be recovered using the inverse transform

$$f(x) = \mathcal{F}^{-1}(F)(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\,\omega x}\,d\omega.$$

Find the Fourier transform of $f(x) = \dfrac{1}{1+x^2}$.

6: For a function $f(t)$ defined for $0 < t \in \mathbf{R}$, the **Laplace transform** of $f(t)$ is the function $F(s) = \mathcal{L}(f)(s)$ defined for all s in a set of the form $\{s \in \mathbf{C} | \mathrm{Re}(s) > c\}$ for some $c \in \mathbf{R}$, by

$$F(s) = \mathcal{L}(f)(s) = \int_0^{\infty} f(t)e^{-st}\,dt.$$

Given $F(s)$ the function $f(t)$ can be recovered using the inverse transform

$$f(t) = \mathcal{L}^{-1}(F)(t) = \frac{1}{2\pi\,i}\lim_{R\to\infty}\int_{\lambda_{a,R}} e^{st}F(s)\,ds\,,$$

where $a > c$ and $\lambda_{a,R}(u) = a + i\,u$ for $-R \le u \le R$.

Find the inverse Laplace transform of $F(s) = \dfrac{2s+3}{s^2+4}$ defined for $s \in \mathbf{C}$ with $\mathrm{Re}(s) > 0$.