ECE 206 Advanced Calculus 2, Assignment 11 Due TBA

1: (a) Let $\alpha(t)=t+i\,e^{it}$ with $0\le t\le 2\pi$. Sketch the path α, then find its length.
(b) Let $\alpha(t)=(2-4\sin t)e^{it}$ for $0\le t\le 2\pi$. Sketch $\alpha(t)$ and use the sketch to find the winding numbers $\eta(\alpha,a_i)$ for $a_1=i$, $a_2=-i$ and $a_3=-3i$.

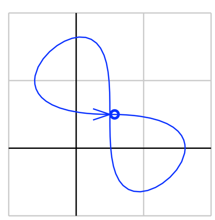
2: (a) Find $\displaystyle\int_0^{2\pi}\sin t\,e^{it}\,dt$
(b) Find $\displaystyle\int_\alpha \frac{e^z\,dz}{(e^z+i)^2}$ where $\alpha(t)=i\,\frac{\pi}{4}\left(1+e^{i\,t}\right)$ for $0\le t\le\pi$.

3: (a) Find $\displaystyle\int_\alpha z^2-2i\,\overline{z}\,dz$ where $\alpha(t)=-i+(1+i)t$ for $0\le t\le 3$.
(b) Find $\displaystyle\int_\alpha z\sqrt{z+1}\,dz$, using the branch of the square root given by $\sqrt{r\,e^{i\theta}}=\sqrt{r}\,e^{i\,\theta/2}$ for $r>0$ and $-\pi<\theta<\pi$, where $\alpha(t)=(1-i)+(2+i)t$ for $-1\le t\le 1$.

4: Find $\displaystyle\int_\alpha \frac{\log(z+i)}{z^2(z-1)}\,dz$ where $\log(z+i)=\ln|z+i|+i\,\theta(z+i)$ for $-\frac{\pi}{2}<\theta(z+i)<\frac{3\pi}{2}$ and $\alpha(t)=(1+i)+2e^{it}$ with $0\le t\le 2\pi$.

5: Find $\displaystyle\int_\alpha \frac{dz}{z(z^2+1)^2}$ where $\alpha(t)=i+\sqrt{2}\,e^{it}$ with $\frac{5\pi}{4}\le t\le\frac{7\pi}{4}$.

6: Find $\displaystyle\int_\alpha \frac{\sin(\pi z)}{z^4-1}\,dz$, where $\alpha(t)=\frac{1}{2}(1+i)+6(t^2-1)t^3+i\,4(t^2-1)^2t$ for $-1\le t\le 1$. The loop α is shown below.



7: Find $\displaystyle\int_\alpha \frac{dz}{z^2(z-1)(z^2-2z+2)}$ where $\alpha(t)=\sin t+2\cos(2t)+i\,\cos(3t)$ with $-\frac{3\pi}{2}\le t\le\frac{\pi}{6}$. The path α is shown below. It starts at the point -1 and ends at the point $\frac{3}{2}$.