ECE 206 Advanced Calculus 2, Assignment 10 Due TBA

1: (a) Let $f(z) = e^{z^2}/z$. Find $f'(i)$ and $f'\left(\frac{\sqrt{\pi}}{2}(1+i)\right)$.

(b) Let $f\left(r\,e^{i\,\theta}\right) = \sqrt{r}\,e^{i\,\theta/2}$ where $r > 0$ and $0 < \theta < 2\pi$. Find $f'(2i)$ and $f'(3-4i)$.

2: A long thin straight wire lies along the z-axis between two grounded plates which lie along the planes $x = \pm 1$. The wire carries a uniformly distributed charge of charge density ρ.

(a) Find the electric potential u at all points between the two plates.

(b) Find the electric field E at all points between the plates.

(c) Find the direction of the electric field E at the point $(x, y, z) = \left(\frac{1}{3}, \frac{\ln 4}{\pi}, 0\right)$.

3: Let f be the inverse of the restriction of $\sin z$ to the set $\left\{x + i\,y \middle| -\frac{\pi}{2} < x < \frac{\pi}{2}\right\}$. Find $f'\left(\frac{4}{3}\,i\right)$ and $f'\left(-\frac{5+3i}{4\sqrt{2}}\right)$.

4: (a) Find the temperature $u(z)$ at each point $z \in \mathbf{C}$ given that the temperature along the positive real axis is held constant with $u(x) = 40$ for $0 \le x < 1$ and $u(x) = 10$ for $x > 1$.

(b) Find the isotherm $u = 25$.

5: One metal plate lies along the surface $\left\{(x, y, z) \middle| x^2 + y^2 = 1, y > 0\right\}$ and another lies along the surface $\left\{(x, y, z) \middle| x^2 + (y+1)^2 = 2, y > 0\right\}$. The first plate is held at a constant potential of 1 and the second is held at a constant potential of 4 (they are separated by insulating material along the lines $x = \pm 1$, $y = 0$).

(a) Find the electric potential u at all points between the two plates.

(b) Find the point $(0, y, 0)$ where $u = 3$.