ECE 206 Advanced Calculus 2, Assignment 1 Due TBA

1: Let $\mathcal{C}$ be the parametric curve $(x, y) = f(t) = (\cos t, \sin 2t)$.

(a) Make an accurate sketch of the curve $\mathcal{C}$.

(b) Find an implicit equation for the tangent line to $\mathcal{C}$ at the point where $t = \frac{\pi}{3}$.

(c) Find an implicit equation for $\mathcal{C}$.

2: Let $\mathcal{S}$ be the parametric surface $(x, y, z) = f(s, t) = \left(\frac{s}{t}, \sqrt{s+t}, st\right)$.

(a) Find the derivative matrix $Df(s, t)$.

(b) Find a parametric equation for the tangent plane to $\mathcal{S}$ at the point where $(s, t) = (2, 2)$.

(c) Find an implicit equation for the tangent plane to $\mathcal{S}$ at the point where $(s, t) = (2, 2)$.

3: Let $\mathcal{C}$ be the curve of intersection of the two surfaces $z = x^2 - 2y$ and $z = 2x^2 + y^2$. Find a parametric equation for the tangent line $\mathcal{L}$ to the curve $\mathcal{C}$ at the point $(-1, -1, 3)$ using each of the following two methods.

(a) Find the equation of the tangent plane to each of the two surfaces at $(-1, -1, 3)$, then solve the two equations to obtain a parametric equation for $\mathcal{L}$.

(b) Find a parametric equation for $\mathcal{C}$, then use this parametric equation to find a parametric equation for the tangent line $\mathcal{L}$.

4: (a) Let $\mathcal{P}$ be the tangent plane to the surface given by $z = 4x^2 - 8xy + 5y^2$ at the point where $(x, y) = (2, 1)$. Find the line of intersection of $\mathcal{P}$ with the xy-plane.

(b) Find the equation of the tangent plane to the surface given by $e^{x+z} = \sqrt{x^2 y + z}$ at the point $(1, 2, -1)$.

5: Let $\mathcal{S}$ be the surface $2yz = x^2 + y^2$.

(a) Sketch the level sets $z = -2, -1, 0, 1, 2$ for the surface $\mathcal{S}$ (in other words, sketch the curve of intersection of $\mathcal{S}$ with the each of the planes $z = -2, -1, 0, 1, 2$).

(b) Sketch the surface $\mathcal{S}$.

(c) Find the equation of the tangent plane to $\mathcal{S}$ at the point $(3, 1, 5)$.

6: The position of a fly at time t is given by $(x, y, z) = (t, t^2, 1 + t^3)$. A light shines down on the fly from the point $(0, 0, 3)$ and casts a shadow on the xy-plane. Find the position and the velocity of the shadow at time $t = 1$.