Name (print): ____________________________

Signature: ____________________________

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CO 250, Introduction to Optimization

Midterm Test, Winter Term, 2013

University of Waterloo, UAE Campus

Instructor: Stephen New

Date: Feb 28, 2013

Time: 9:00 – 10:20 am

Instructions:

1. Place your name, signature and ID number in the spaces provided at the top of this page.

2. This test contains 7 pages, including this cover page and a page at the end for rough work.

3. No calculators are allowed.

4. Answer all 5 questions; all questions will be given equal value.

5. Provide full explanations with all your solutions.

Question	Mark
1	/5
2	/5
3	/5
4	/5
5	/5
Total	/25

[5] **1:** Maximize and minimize $z = c_0 + c^T x$ for $x \in \mathbf{R}^5$ subject to $Ax = b$ and $x \geq 0$, where

$$c_0 = -5 \ , \ c = \big(1, 2, 0, 3, 1\big)^T \ , \ A = \begin{pmatrix} 1 & 0 & 1 & -1 & -2 \\ 2 & 1 & 2 & -1 & -3 \\ 2 & -1 & 1 & -4 & -2 \end{pmatrix} \ , \ b = \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix} .$$

[5] **2:** Maximize and minimize $w = 3x - y + z$ subject to $x + z = 2$, $x^2 + y^2 \leq 5$ and $2x \leq y\,z$.

[5] **3:** Consider the LP where we *minimize* $z = 4x_1 + 3x_2 - 2x_3$ subject to $x_1 + 2x_2 - x_3 = 1$, $3x_1 + x_2 - 2x_3 \geq 4$, $-2x_1 - x_2 + x_3 \leq -2$, $x_1 \geq 0$ and $x_2 \leq 0$.

(a) Convert the given LP into an equivalent LP in SEF for $\widetilde{x} = \left(x_1, {x_2}^-, {x_3}^+, {x_3}^-, s\,, t\,\right)^T$. Express the answer in matrix form.

(b) Given that $\overline{x} = (x_1, x_2, x_3)^T = (0, -1, -3)^T$ is an optimal solution to the the given LP, find an optimal solution to the equivalent LP in SEF that you obtained in part (a).

[5] **4:** Consider an LP in SEF with constraints $Ax = b$ and $x \geq 0$, where

$$A = \begin{pmatrix} 1 & 0 & 0 & -2 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & -3 \end{pmatrix}, \ b = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}.$$

Show that the LP is unfeasible and find a certificate of unfeasibility.

[5] **5:** Consider the LP where we maximize $z(x) = c_0 + c^T x$ subject to $Ax = b$ and $x \geq 0$ where

$$c_0 = 2\ ,\ c = \big(0,0,1,0,0,2\big)^T\ ,\ A = \begin{pmatrix} -1 & 1 & 1 & 0 & 0 & 1 \\ -1 & 0 & 2 & 1 & 0 & -1 \\ -3 & 0 & 1 & 0 & 1 & 2 \end{pmatrix}\ ,\ b = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}.$$

(a) Use the Simplex Algorithm, starting with the feasible basis $\mathcal{B} = \{2,4,5\}$, to show that the LP is unbounded.

(b) Find a feasible point x with $z(x) = 100$.

This page is for rough work. It will not be marked.