Name (print): ______________________

Signature: ______________________

ID Number: ______________________

CO 250, Introduction to Optimization

Midterm Test, Winter Term, 2012

University of Waterloo, UAE Campus

Instructor: Stephen New

Date: March 1, 2012

Time: 9:00 – 10:20 am

Question	Mark
1	/5
2	/5
3	/5
4	/5
5	/5
6	/5
Total	/30

Instructions:

1. Place your name, signature and ID number in the spaces provided at the top of this page.

2. This test contains 8 pages, including this cover page and a page at the end for rough work.

3. No calculators are allowed.

4. Answer all 6 questions; all questions will be given equal value.

5. Provide full explanations with all your solutions.

[5] **1:** Maximize and minimize $z = c^T x$ for $x \in \mathbf{R}^5$ subject to $Ax = b$ and $x \geq 0$, where

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & -1 & -1 \end{pmatrix} \ , \ b = \begin{pmatrix} 5 \\ 5 \\ -1 \end{pmatrix} \ , \ c = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \\ 3 \end{pmatrix} .$$

You may solve this using a sketch of the feasibility set.

[5] **2:** A company produces 2 products, P_1 and P_2. Production of each product requires the use of 3 resources R_1, R_2 and R_3. Let a_{ij} be the number of units of R_i needed to produce one unit of P_j, let c_i be the cost per unit of R_i, let m_i be the maximum number of units of R_i available to the company, and let p_j be the selling price per unit of P_j. Suppose these constants are as follows.

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 1 & 1 \end{pmatrix} \ , \ c = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \ , \ m = \begin{pmatrix} 8 \\ 12 \\ 5 \end{pmatrix} \ , \ p = \begin{pmatrix} 10 \\ 9 \end{pmatrix} .$$

Set up an LP which could be solved to determine the number of units x_j of each product P_j the company should produce to maximize its profit (you do not need to solve the LP).

[5] **3:** Consider the LP where we maximize $z = 5 + 3x_1 + x_2 - 2x_3$ subject to $x_1 - 2x_2 + x_3 \geq 1$, $x_1 - x_2 + 3x_3 \leq 4$ and $2x_1 - x_3 = 3$. Convert this LP to an LP in SEF and give the tableau for the new LP.

[5] **4:** Let $A = \begin{pmatrix} 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & 1 & -3 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $c = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 2 \end{pmatrix}$.

(a) Determine whether $\overline{x} = (1,4,3,2,2)^T$ and $y = (0,1,2,1,1)^T$ form a certificate of unboundedness for the LP where we maximize $z = c^T x$ subject to $Ax = b$ and $x \geq 0$.

(b) Determine whether $\overline{x} = (0,5,0,2,1)^T$ and $y = \left(\frac{1}{2}, 0, \frac{1}{2}\right)^T$ form a certificate of optimality for the LP where we maximize $w = -z = (-c)^T x$ subject to $Ax = b$ and $x \geq 0$.

[5] **5:** Use the Simplex Algorithm, starting with the feasible basis $\mathcal{B} = \{1, 4, 5\}$, to find the vector $x \in \mathbf{R}^6$ which maximizes $z = c_0 + c^T x$ subject to $Ax = b$ and $x \geq 0$ where $c_0 = 1$ and

$$A = \begin{pmatrix} 1 & 2 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 & 0 & -1 \\ 0 & 2 & -1 & 0 & 1 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}, \quad c = \begin{pmatrix} 0 \\ 3 \\ -1 \\ 0 \\ 0 \\ 2 \end{pmatrix}.$$

[5] **6:** Consider the LP with tableau

$$\begin{pmatrix} -c^T & c_0 \\ A & b \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 & -2 & 0 & -6 & 3 \\ -1 & 0 & 1 & 1 & 0 & 3 & 2 \\ 3 & 0 & 0 & 2 & 1 & 2 & 1 \\ 2 & 1 & 0 & -1 & 0 & 1 & 0 \end{pmatrix}.$$

Note that, up to a permutation of the rows, the LP is in canonical form for $\mathcal{B} = \{2, 3, 5\}$.

(a) If we pivot at position $(1, 4)$, then what will the new canonical basis be?

(b) If we pivot at position $(2, 6)$, then will the new basis be feasible?

(c) If we pivot at position $(1, 1)$, then what will the new basic point be?

(d) If we pivot at position $(2, 1)$, then what will be the value of z at the new basic point?

(e) If we pivot at position $(1, 6)$, then will the value of z at the new basic point be optimal?

This page is for rough work. It will not be marked.