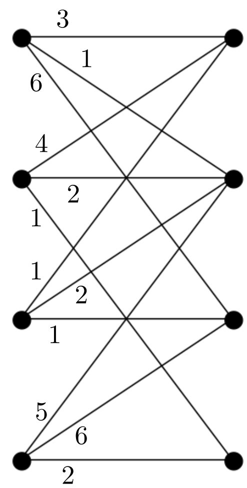
1: Recall that we formalized the maximum weight perfect matching problem using the following LP. Given a weighted graph G, we introduce variables x_e for each edge $e \in E$, and we maximize $z = \sum_{e \in E} c_e x_e$ where $c_e = \text{weight}(e)$ subject to $\sum_{e \in E, v \in e} x_e = 1$ for each vertex v and $x_e \geq 0$ for each edge e. Using Phases I and II of the Simplex Algorithm to solve this LP, and using our formula for a certificate of optimality, find a maximum weight perfect matching and an optimal dual solution for the weighted graph G with vertex set $V = \{v_1, v_2, v_3, v_4\}$, edge set $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ where $e_1 = \{v_1, v_2\}$, $e_2 = \{v_1, v_3\}$, $e_3 = \{v_1, v_4\}$, $e_4 = \{v_2, v_3\}$, $e_5 = \{v_2, v_4\}$, $e_6 = \{v_3, v_4\}$, and weight vector $c = \big(2, 1, 4, 3, 5, 3\big)^T$.

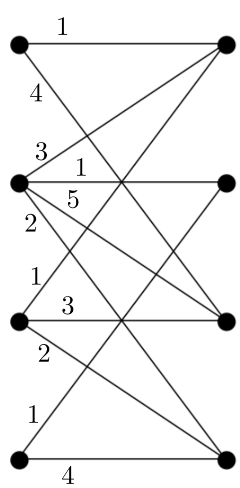
2: Consider the weighted graph shown below.



(a) Use the Maximum Weight Perfect Matching Algorithm to find a maximum weight perfect matching and and optimal dual solution.

(b) Use the Maximum Weight Matching Algorithm to find a maximum weight matching and and optimal dual solution.

3: Consider the weighted graph shown below.



(a) Use the Maximum Weight Perfect Matching Algorithm to find a maximum weight perfect matching and and optimal dual solution.

(b) Use the Maximum Weight Matching Algorithm to find a maximum weight matching and and optimal dual solution.

4: Consider the IP where we maximize $z = c^T x$ subject to $Ax = b$ and $x \geq 0$, where

$$A = \begin{pmatrix} -1 & 1 & 3 & -1 \\ -2 & 1 & 4 & -3 \end{pmatrix} \ , \ b = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \ , \ c = \big(-1, 1, 4, 2 \big)^T .$$

Solve the IP, using the Simplex Algorithm repeatedly, beginning with the LP relaxation of the given IP and then finding cutting planes and adding the corresponding inequality constraints.

5: Given a graph G, the *Maximum Set of Isolated Vertices Problem* is to find a set of vertices $S \subseteq V(G)$ of largest possible size such that no two vertices in S are endpoints of the same edge in G.

(a) Formulate the Maximum Set of Isolated Vertices Problem as an IP (introducing an integer variable x_v for each vertex v). Express your answer in matrix form.

(b) Find and simplify the DLP (that is the dual of the LP relaxation of the IP).

(c) Determine the duality gap in the case of the *complete graph* on n vertices, that is the graph with n vertices and $\binom{n}{2}$ edges (so there is an edge joining every pair of vertices).