CO 250 Intro to Optimization, Assignment 3 — Due Tues Feb 26

1: Consider an LP with constraints $Ax = b$ and $x \geq 0$ where

$$A = \begin{pmatrix} 1 & 1 & 2 & -1 & -4 & 0 \\ 1 & 0 & -1 & 1 & 1 & -2 \\ 2 & 1 & 3 & -1 & -4 & -3 \\ 1 & 2 & 2 & 0 & -3 & 2 \end{pmatrix} , \; b = \begin{pmatrix} -1 \\ 2 \\ -1 \\ 8 \end{pmatrix} .$$

Use a picture of the feasible set to find every feasible basis for the LP and all of the corresponding feasible basic points.

2: Consider the LP where we maximize $z(x) = c_0 + c^T x$ subject to $Ax = b$, $x \geq 0$ where

$$c_0 = 6 \; , \; c = \big(3, 1, -2, 1, 2, -5\big)^T \; , \; A = \begin{pmatrix} 2 & 1 & -1 & 2 & 1 & 0 \\ 4 & 2 & -2 & 5 & 3 & 1 \\ 3 & 1 & -2 & 4 & 4 & -1 \end{pmatrix} \; , \; b = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} .$$

Put the LP into canonical form for the basis $B = \{2, 4, 5\}$ in the following two ways.

(a) Use row operations on the tableau $\begin{pmatrix} -c^T & c_0 \\ A & b \end{pmatrix}$ to obtain the tableau $\begin{pmatrix} -\widetilde{c}^T & \widetilde{c}_0 \\ \widetilde{A} & \widetilde{b} \end{pmatrix}$.

(b) Calculate A_B^{-1} then use the formulas $\widetilde{A} = A_B^{-1} A$, $\widetilde{b} = A_B^{-1} b$ and $\widetilde{c}_0 = c_0 + b^T y$ and $\widetilde{c} = c - A^T y$ where $y = A_B^{-T} c_B$.

3: Consider the LP where we maximize $z(x) = c_0 + c^T x$ subject to $Ax = b$, $x \geq 0$, where

$$c_0 = 2 \; , \; c = \big(1, 1, -1, 1, -5\big)^T \; , \; A = \begin{pmatrix} 1 & 2 & -1 & -4 & -2 \\ 2 & 1 & -1 & -1 & -6 \\ -1 & 1 & 1 & -2 & 5 \end{pmatrix} \; , \; b = \begin{pmatrix} -4 \\ -9 \\ 10 \end{pmatrix} .$$

Let $B = \{2, 3, 5\}$, $B' = \{2, 4, 5\}$ and $B'' = \{1, 2, 4\}$. By performing row operations on the tableau, find the basic points u, u', u'' for these bases, find the values $z(u)$, $z(u')$ and $z(u'')$, and find an optimal solution to the LP.

4: Consider the LP where we maximize $z(x) = c_0 + c^T x$ subject to $Ax = b$, $x \geq 0$, where

$$c_0 = 3 \; , \; c = \big(1, -2, 1, 3, -1\big)^T \; , \; A = \begin{pmatrix} 1 & 2 & 1 & 3 & -4 \\ 2 & 1 & 1 & 0 & -4 \\ 1 & -3 & 1 & -2 & 1 \end{pmatrix} \; , \; b = \begin{pmatrix} 5 \\ 2 \\ 0 \end{pmatrix} .$$

Use Phase II of the Simplex Algorithm, starting with the feasible basis $B = \{2, 3, 5\}$, to solve the LP.

5: Consider the LP where we maximize $z(x) = c_0 + c^T x$ subject to $Ax = b$, $x \geq 0$, where

$$c_0 = -2 \; , \; c = \big(-2, -4, -1, 1, 4, 3\big)^T \; , \; A = \begin{pmatrix} 1 & 5 & 2 & -1 & -1 & 0 \\ 2 & 0 & -3 & 1 & -3 & -1 \\ 3 & 4 & -1 & 1 & -2 & 1 \end{pmatrix} \; , \; b = \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix} .$$

Use Phase II of the Simplex Algorithm, starting with the feasible basis $B = \{1, 3, 4\}$, to solve the LP.