CO 250 Intro to Optimization, Assignment 1 Due Tues Jan 22

1: Maximize and minimize $z = c^T x$ for $x \in \mathbf{R}^5$ subject to $Ax = b$ and $x \geq 0$ where

$$c = \big(3, 1, -2, -5, 3\big)^T \ , \ A = \begin{pmatrix} 1 & 2 & 1 & -2 & -3 \\ 1 & 3 & 2 & -2 & -5 \\ 3 & 1 & -1 & -5 & 2 \end{pmatrix} \ , \ b = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix} .$$

2: Maximize and minimize $z = c^T x$ for $x \in \mathbf{Z}^4$ (this is an IP) subject to $Ax = b$, $x \geq 0$ where

$$c = \big(1, 2, -1, 0\big)^T \ , \ A = \begin{pmatrix} 1 & 2 & 0 & -2 \\ 3 & 2 & 2 & 0 \end{pmatrix} \ , \ b = \begin{pmatrix} -3 \\ 5 \end{pmatrix} .$$

3: Maximize and minimize $w = x + y + z$ for $(x, y, z)^T \in \mathbf{R}^3$ subject to the non-linear constraints

$$x + 2y - 2z = 1 \ , \ 3x + y^2 + z^2 - 6z \leq 4 \ , \ 3x + 5y - z^2 \geq 8 \, .$$

4: Let $A, B, C, u, v \geq 0$.

(a) Suppose we wish to maximize $z = c^T x$ for $x \in \mathbf{R}^n$ subject to the condition that $x \geq 0$ and either $Ax \geq u$ or $Bx \geq v$. Show that this problem can be formulated as an IP.

(b) Suppose that we wish to maximize $z = c^T x$ for $x \in \mathbf{R}^n$ subject to the condition that $x \geq 0$ and at least two of the three matrix inequalities $Ax \geq u$, $Bx \geq v$ and $Cx \geq w$ are satisfied. Show that this problem can be formulated as an IP.

5: In Conway's game of life, we are given an $n \times n$ grid with cells labeled by pairs (k, l) with $1 \leq k \leq n$ and $1 \leq l \leq n$. Each cell has at most 8 neighbouring cells, where the neighbours of the cell (k, l) are the cells $(k \pm 1, l), (k, l \pm 1), (k \pm 1, l \pm 1)$. Each cell can be either alive or dead. The initial set of living cells is denoted by $L = L_0$. At each stage in the game, the set of living cells changes giving sets $L_0, L_1, L_2, \cdots$. The set L_{n+1} is determined from the set L_n as follows. For each cell (k, l),

if there is at most 1 neighbour of the cell (k, l) which lies in L_n then $(k, l) \notin L_{n+1}$,
if there are exactly 2 neighbours of (k, l) in L_n then $(k, l) \in L_{n+1} \iff (k, l) \in L_n$,
if there are exactly 3 neighbours of (k, l) in L_n then $(k, l) \in L_{n+1}$, and
if there are at least 4 neighbours of (k, l) in L_n then $(k, l) \notin L_{n+1}$.

Suppose that we are given a positive integer n and we wish to find the largest possible size for a set $L = L_0$ with the property that $L_0 = L_1 = L_2 = \cdots$. Show that this problem can be formulated as an IP.