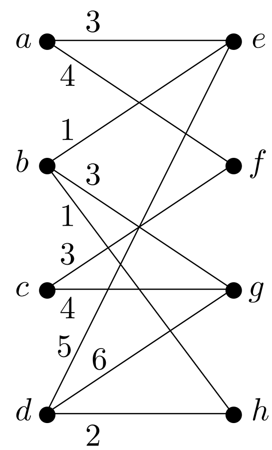
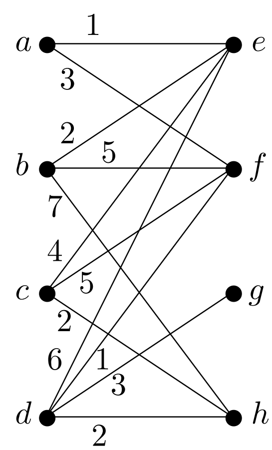
CO 250 Intro to Optimization, Assignment 6 Not to be handed in

1: Recall that we formalized the maximum weight perfect matching problem using the following LP. Given a weighted graph G, we introduce variables x_e for each edge $e \in E$, and we maximize $z = \sum_{e\in E} c_e x_e$ where $c_e = \text{weight}(e)$ subject to $\sum_{e\in E \text{ s.t. } v\in e} x_e = 1$ for each vertex v and $x_e \geq 0$ for each edge e. Using the Simplex Algorithm to solve this LP, and using our formula for a certificate of optimality, find a maximum weight perfect matching and an optimal dual solution for the weighted graph G with vertex set $V = \{a, b, c, d\}$, edge set $E = \{ab, ac, bc, bd\}$ and weights $c = \big(c_{ab}, c_{ac}, c_{bc}, c_{bd}\big)^T = (5, 4, 6, 3)^T$.

2: (a) Use the Hungarian Algorithm to find a maximum weight perfect matching in the following weighted graph G.



(b) Use the Hungarian Algorithm to find a maximum weight matching in the following weighted graph G.



3: Consider the IP where we maximize $z = c^T x$ subject to $Ax = b$ and $x \geq 0$, where

$$A = \begin{pmatrix} 1 & 1 & 2 & 5 \\ 1 & -3 & -2 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 12 \\ -4 \end{pmatrix}, \quad c = \big(-2, 5, 4, -1\big)^T.$$

(a) Find the duality gap for this IP by solving both the IP and its LP relaxation using an accurate sketch of the feasible set.

(b) Solve the LP relaxation using the Simplex Algorithm beginning with the feasible basis $\mathcal{B} = \{1, 2\}$, find a cutting-plane and add the corresponding inequality to the constraints, put the new LP into SEF and solve it using the Simplex Algorithm, beginning with a sensibly chosen feasible basis.