CO 250 Intro to Optimization, Assignment 5 Due Sun Apr 1

1: Consider the LP (not in SEF) where we maximize $z = c^T x$ subject to $Ax \le b$ and $x \ge 0$.

(a) Put the LP into SEF then find and simplify the DLP.

(b) Find optimal solutions $\overline{x}$ and $\overline{y}$ to the LP and the DLP when

$$A = \begin{pmatrix} 2 & 5 & 3 \\ 1 & 2 & 1 \\ 2 & 3 & 4 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 10 \\ 5 \end{pmatrix} \text{ and } c = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}.$$

2: Consider the LP where we maximize $z = c^T x$ subject to $Ax = b$ and $x \ge 0$ where

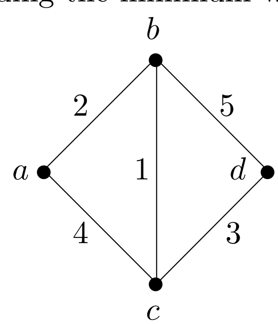
$$A = \begin{pmatrix} 1 & 4 & 2 & 15 & 2 & 0 & 7 \\ 0 & 1 & 1 & 6 & 1 & 0 & 3 \\ -1 & 1 & 1 & 6 & 2 & 1 & 6 \\ -5 & -8 & 3 & 2 & 3 & -1 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 7 \\ 3 \\ 4 \\ 2 \end{pmatrix} \text{ and } c = \big(-1, 2, -2, -5, 3, -1, -2\big)^T.$$

Let $\overline{x} = (1, 0, 3, 0, 0, 2, 0)^T$.

(a) Show that $\overline{x}$ is a basic feasible solution for the basis $\mathcal{B} = \{1, 3, 5, 6\}$ but that the vector $y = {A_{\mathcal{B}}}^{-T} c_{\mathcal{B}}$ is not a certificate of optimality for $\overline{x}$.

(b) Show that $\overline{x}$ is an optimal solution and find a certificate of optimality for $\overline{x}$.

3: Let G be the weighted graph with vertex set $V = \{a, b, c, d\}$, edge set $E = \{ab, ac, bc, bd, cd\}$, and weights given by the vector $c = (2, 4, 1, 5, 3)^T$ (so for example $w(ab) = 2$ and $w(ac) = 4$). Consider the problem of finding the minimum weight path in G from a to d.



(a) Let $M = \big\{ S \subseteq V \big| a \in S, d \notin S \big\} = \big\{ \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\} \big\}$. Find cut$(S)$ for each $S \in M$ then find the matrix A with entries

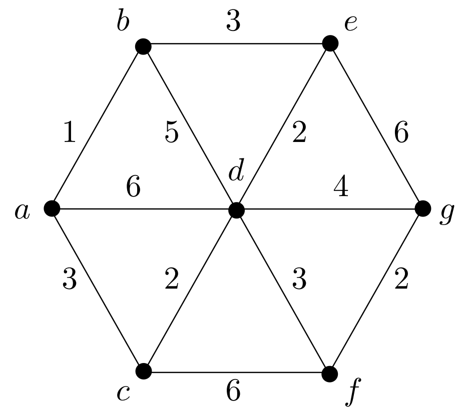
$$A_{S,e} = \begin{cases} 1 \text{ if } e \in \text{cut}(S) \\ 0 \text{ if } e \notin \text{cut}(S). \end{cases}$$

(b) The minimum weight path problem is to minimize $z = c^T x$ subject to $Ax \ge \mathbb{1}$, $x \ge 0$. Put this LP into SEF, find and simplify the DLP, then put the DLP into SEF.

(c) Solve the DLP using the simplex algorithm.

(d) Use your solution from part (c) to find an optimal solution to the LP.

4: Consider the problem of finding the minimum weight path from a to g in the weighted graph G shown below.



(a) Let u be the vector with entries u_S for each $S \in M = \big\{S \subset V \big| a \in S, g \notin S\big\}$ whose non-zero entries are

S	$\{a\}$	$\{a,b,c\}$	$\{a,b,c,d\}$	$\{a,b,c,d,e,f\}$
u_S	1	1	2	2

Determine whether u is a feasible dual solution and, if so, whether u is optimal.

(b) Use the algorithm from class to find a minimum weight path in G from a to f along with an optimal dual solution.

5: Consider the following problem. Given a graph G with vertex set V and edge set E, we wish to find the vector x, with entries x_e for each $e \in E$, which maximizes the sum $z = \sum_{e \in E} x_e$ subject to the constraints $x_e \geq 0$ for all $e \in E$ and $\sum_{e \in E, v \in e} x_e \leq 2$ for all $v \in V$.

(a) Find A, b and c so that the problem is to maximize $c^T x$ subject to $Ax \leq b$ and $x \geq 0$, then put the LP into SEF.

(b) Show that the complementary slackness conditions are that

(1) $\sum_{v \in e} y_v = 1$ for each $e \in E$ with $x_e \neq 0$, and

(2) $y_v = 0$ for each $v \in V$ with $\sum_{e \in E, v \in e} x_e < 2$.

(c) For the graph G shown below, find an optimal solution x with $x_e \in \{0,1\}$ for all $e \in E$, and an optimal dual solution y with $y_v \in \{0,1\}$ for all $v \in V$.