CO 250 Intro to Optimization, Assignment 4 Due Thurs Mar 15

1: (a) Consider the LP (not in SEF) where we maximize $z = 3x - y - 2$ for $0 \leq x, y \in \mathbf{R}$ with $x - 2y \leq 1$, $x - y \leq 2$, $3x - 2y \leq 7$ and $x + y \leq 4$. Put this LP into SEF then aply the Simplex Algorithm, beginning with the obvious feasible basis, to solve the LP.

(b) Consider the LP where we maximize $z = c^T x$ for $x \in \mathbf{R}^6$ with $Ax = b$ and $x \geq 0$, where

$$A = \begin{pmatrix} 1 & 2 & 1 & 0 & 1 & 3 \\ 2 & 3 & 3 & 1 & 2 & 5 \\ 2 & 5 & 2 & 1 & 3 & 6 \end{pmatrix}, \; b = \begin{pmatrix} 3 \\ 7 \\ 6 \end{pmatrix} \text{ and } c = \big(0, 1, 1, 4, 1, 2\big)^T.$$

Use the Simplex Algorithm, starting with the feasible basis $\mathcal{B} = \{1, 2, 3\}$, to solve the LP.

2: Consider the LP where we maximize $z = c^T x$ subject to $Ax = b$ and $x \geq 0$ where

$$A = \begin{pmatrix} 1 & 2 & -2 & 1 & 0 & 2 \\ 2 & 4 & -2 & 3 & 1 & 3 \\ 1 & 3 & -1 & 2 & 1 & 1 \end{pmatrix}, \; b = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} \text{ and } c = \big(1, -1, -2, 3, 2, 3\big)^T.$$

(a) Use Phase I of the Simplex Algorithm to show that the LP is feasible and to find a feasible basis.

(b) Beginning with the feasible basis found in part (a), apply Phase II of the Simplex Algorithm to solve the LP.

3: Consider the LP in SEF with tableau $\begin{pmatrix} -c^T & c_0 \\ A & b \end{pmatrix}$.

(a) Suppose that we appply the Simplex Algorithm to the LP and obtain a basis $\mathcal{B}$ whose basic solution $\overline{x}$ maximizes z. Show that $\overline{x}$ together with the vector $y = {A_{\mathcal{B}}}^{-T} c_{\mathcal{B}}$ form a certificate of optimality for the LP.

(b) Suppose that we apply the Simplex Algorithm to the LP, ending with the modified LP with tableau $\begin{pmatrix} -\widetilde{c}^T & \widetilde{c}_0 \\ \widetilde{A} & \widetilde{b} \end{pmatrix}$ in canonical form for the basis $\mathcal{B}$, with $\widetilde{c}_k > 0$ and $\widetilde{A}_k \leq 0$ (so that the original LP is unbounded). Show that the basic solution $\overline{x}$ for $\mathcal{B}$ together with the vector y given by $y_{\mathcal{B}} = -\widetilde{A}_k$ and $y_{\mathcal{N}} = (e_k)_{\mathcal{N}}$ form a certificate of unboundedness for the original LP.

(c) Suppose that we apply Phase I of the Simplex Algorithm by solving the auxiliary LP in which we maximize $w = -\sum s_i$ subject to $Ax + s = b$ with $x \geq 0$ and $s \geq 0$, and we obtain an optimal solution $\begin{pmatrix} \overline{x} \\ \overline{s} \end{pmatrix}$ with $w = w_{\max} < 0$ (so the original LP is unfeasible). Show that if y is a certificate of optimality for the optimal solution for the auxiliary LP, then the same vector y is also a certificate of unfeasibility for the original LP.

4: Consider the LP where we maximize $z = c^T x$ subject to $Ax = b$ and $x \geq 0$ where

$$A = \begin{pmatrix} 1 & 2 & 3 & 1 & 1 & 4 \\ 1 & 1 & 2 & 0 & 1 & 3 \\ 2 & 3 & 4 & 2 & 1 & 5 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \text{ and } c = \big(2, 1, 3, 0, 1, 4\big)^T .$$

(a) Use Phase I of the Simplex Algorithm, to show that the LP is unfeasible.

(b) Use the results of problem 3 to obtain a certificate of unfeasibility for the LP.

5: (a) Consider the LP where we maximize $z = 2x_1 - x_2 + 3x_3$ for $x_1, x_2, x_3 \in \mathbf{R}$ subject to the constraints $x_1 + 3x_2 - x_3 \geq -1$, $2x_1 + x_2 - 4x_3 \leq 3$, $x_1 + 2x_2 + x_3 = 1$ and $x_3 \geq 0$. Put the LP into SEF using the variables ${x_1}^+, {x_1}^-, {x_2}^+, {x_2}^-, x_3, s_1, s_2$, then find and simplify the dual LP.

(b) Consider the LP where we maximize $z = c^T x$ for $x \in \mathbf{R}^2$ subject to $Ax \leq b$ where

$$A = \begin{pmatrix} 1 & 1 \\ -1 & 2 \\ 2 & -3 \\ -3 & -1 \\ -1 & -1 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 5 \\ 7 \\ 8 \\ 4 \end{pmatrix} \text{ and } c = \begin{pmatrix} -1 \\ -3 \end{pmatrix} .$$

Put the LP into SEF, find and simplify the dual LP, then put the dual LP into SEF.