CO 250 Intro to Optimization, Assignment 3 Due Thurs Feb 16

1: Consider an LP with constraints $Ax = b$, $x \geq 0$ where

$$A = \begin{pmatrix} 1 & 0 & 1 & -3 & 2 \\ 2 & 1 & 1 & -2 & -1 \\ 1 & 1 & -1 & 2 & -6 \end{pmatrix} \quad \text{and } b = \begin{pmatrix} 1 \\ 3 \\ -3 \end{pmatrix} .$$

Find the first ordered triple in the list

$$(1,2,3),(1,2,4),(1,2,5),(1,3,4),(1,3,5),(1,4,5),(2,3,4),(2,3,5),(2,4,5),(3,4,5)$$

which is a feasible basis for the LP.

2: Consider the LP in which we maximize $z = c_0 + c^T x$ subject to $Ax = b$ and $x \geq 0$ where

$$c_0 = 4 \ , \ c^T = \big(1,1,2,1,-1,3\big) \ , \ A = \begin{pmatrix} 1 & 2 & 1 & 3 & -1 & 5 \\ 2 & 5 & 3 & 4 & -1 & 7 \\ 1 & 3 & 1 & 2 & 0 & 3 \end{pmatrix} \ , \ b = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} .$$

Put this LP into canonical form for the basis $\mathcal{B} = \{3,4,6\}$ in the following two ways.

(a) Perform row operations on the tableau $\begin{pmatrix} -c^T & c_0 \\ A & b \end{pmatrix}$ to obtain $\begin{pmatrix} -\widetilde{c}^T & \widetilde{c}_0 \\ \widetilde{A} & \widetilde{b} \end{pmatrix}$.

(b) Calculate ${A_{\mathcal{B}}}^{-1}$ then find $\widetilde{A}$, $\widetilde{b}$, $\widetilde{c}_0$ and $\widetilde{c}$ using the formulas $\widetilde{A} = {A_{\mathcal{B}}}^{-1}A$, $\widetilde{b} = {A_{\mathcal{B}}}^{-1}b$, and $\widetilde{c}_0 = c_0 - y^T b$ and $\widetilde{c} = c + A^T y$ where $y = -{A_{\mathcal{B}}}^{-T} c_{\mathcal{B}}$.

3: Consider the LP in which we maximize $z = c^T x$ subject to $Ax = b$ and $x \geq 0$ where

$$A = \begin{pmatrix} 1 & 0 & 2 & -1 & -3 \\ 2 & 1 & 2 & -3 & -1 \\ 1 & -1 & 3 & -1 & -6 \end{pmatrix} \ , \ b = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \ \text{and } c = \big(2,1,3,0,-4\big)^T .$$

Let $\mathcal{B} = \{1,2,3\}$, $\mathcal{B}' = \{1,2,4\}$, $\mathcal{B}'' = \{1,3,4\}$ and $\mathcal{B}''' = \{1,4,5\}$. Find the basic points $\overline{x}$, $\overline{x}'$, $\overline{x}''$ and $\overline{x}'''$ corresponding to these bases, then find the values $z(\overline{x})$, $z(\overline{x}')$, $z(\overline{x}'')$ and $z(\overline{x}''')$, and determine the optimal solution to the given LP.

4: Consider the LP with tableau

$$\begin{pmatrix} -c^T & c_0 \\ A & b \end{pmatrix} = \begin{pmatrix} 0 & -c_2 & -c_3 & 0 & -c_5 & 0 & c_0 \\ 1 & a_{12} & a_{13} & 0 & a_{15} & 0 & b_1 \\ 0 & a_{22} & a_{23} & 1 & a_{25} & 0 & b_2 \\ 0 & a_{32} & a_{33} & 0 & a_{35} & 1 & b_3 \end{pmatrix} .$$

Note that this LP is in canonical form for the basis $\mathcal{B} = \{1,4,6\}$. Suppose that $a_{33} \neq 0$ and let $\mathcal{B}' = \{1,3,4\}$.

(a) Find the 4×4 matrix E such that when this LP is put into canonical form for the basis $\mathcal{B}'$, it has tableau $E \begin{pmatrix} -c^T & c_0 \\ A & b \end{pmatrix}$.

(b) Find necessary and sufficient conditions on a_{ij}, b_i and c_i in order that $\mathcal{B}'$ is feasible.

5: Consider the LP with tableau

$$\begin{pmatrix} -1 & 0 & -2 & -5 & 0 & -4 & 6 & 0 & 0 & -2 & 0 & 3 \\ \\ 2 & 0 & 3 & 2 & 0 & 1 & 2 & 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & -1 & -3 & 0 & -1 & 0 & 0 & 0 & 4 & -2 & 0 \\ 1 & 0 & -2 & 4 & 0 & 3 & 1 & 0 & 1 & 1 & 1 & 2 \\ -2 & 0 & 1 & -2 & 1 & 0 & -3 & 0 & 0 & 3 & -3 & 0 \end{pmatrix}.$$

Note that, up to a permutation of the rows, the LP is in canonical form for the basis $\mathcal{B} = \{2, 5, 8, 9\}$.

(a) For which choices of $k \notin \mathcal{B}$ and $l \in \mathcal{B}$ is $\mathcal{B}' = \big(\mathcal{B} \cup \{k\}\big) \setminus \{l\}$ a basis for the given LP?

(b) For which of the choices in part (a) is the new basis $\mathcal{B}'$ feasible?

(c) For which of the choices in part (b) do we have $z(\overline{x}) = z(\overline{x}')$ (where $\overline{x}$ is the basic point for $\mathcal{B}$ and $\overline{x}'$ is the basic point for $\mathcal{B}'$)?

(d) For which of the choices in part (b) do we have $z(\overline{x}\,') > z(\overline{x})$?

(e) Which of the choices in part (d) gives the maximum increase $\Delta z = z(\overline{x}') - z(\overline{z})$.