CO 250 Intro to Optimization, Assignment 1 Due Thurs Jan 19

In this assignment, problems 1-4 can be solved using an accurate sketch of the feasible set.

1: Maximize and minimize $z = c^T x$ for $x \in \mathbf{R}^5$ subject to $Ax = b$ and $x \geq 0$, where

$$c = \begin{pmatrix} 1 \\ -2 \\ 1 \\ 3 \\ -2 \end{pmatrix}, \ A = \begin{pmatrix} 1 & 4 & -3 & -2 & 3 \\ 1 & 3 & -2 & -1 & 2 \\ 1 & 2 & -2 & -3 & 3 \end{pmatrix}, \text{ and } b = \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix}.$$

2: Maximize and minimize $z = 2x - y$ for $x, y \in \mathbf{Z}$ (this is an integer program) subject to $x + 3y \leq 7$, $3x - 2y \leq 1$ and $5x + 2y \geq -2$.

3: Maximize and minimize $z = x - 2y$ for $x, y \in \mathbf{R}$ subject to the non-linear constraints $x^2 + y^2 \leq 4x + 2y$ and $4y \leq x + xy$.

4: A company produces two products P_1 and P_2 which use two resources R_1 and R_2. They use 2 units of R_1 per unit of P_1 produced and 3 units of R_1 per unit of P_2 produced, and they have a total of 25 units of R_1 available. They use 1 unit of R_2 per unit of P_1 and 2 units of R_2 per unit of P_2, and they have a total of 16 units of R_2 available. They make a profit of 5 thousand dollars per unit of P_1 produced and 8 thousand dollars per unit of P_2 produced. How much would it be worth to purchase 4 more units of R_1?

5: Let P be the polygon $P = \{x \in \mathbf{R}^2 \,|\, Ax \leq b\}$ where $A \in M_{n\times 2}(\mathbf{R})$ and $b \in \mathbf{R}^n$. Show that the problem of determining the maximum possible radius r for a circular disc which is contained in P can be formulated as a linear programming problem.