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## Relative strength of propositional calculi

Cook and Rechkow (1974/1979) looked at the question of whether or not there exists a sound propositional calculus $\mathcal{P}$ with an adequate set of connectives such that tautologies have polynomial length derivations, i.e., such that there is a polynomial $p(x)$ with the property that every tautology $\tau$ has a derivation $\Delta$ for which the total number of occurrences of symbols in $\Delta$ is no more than $p(|\tau|)$, $|\tau|$ being the length of the string $\tau$. Such a $\mathcal{P}$ is said to be *polynomially bounded.* (If a polynomially bounded propositional calculus exists then TAUT is in NP, and hence NP = co-NP.) In an effort to study the above question they introduced the notion of $p$-simulation (polynomial time simulation). A propositional calculus $\mathcal{P}_1$ *p-simulates* another propositional calculus $\mathcal{P}_2$ if there is a polynomial time translation from formulas of $\mathcal{P}_2$ to formulas of $\mathcal{P}_1$ and a polynomial time algorithm to convert derivations in $\mathcal{P}_2$ into derivations of the translates in $\mathcal{P}_1$. If $\mathcal{P}_1$ and $\mathcal{P}_2$ $p$-simulate each other then we say they are *p-equivalent*. Cook and Rechkow focused on variations of well-known propositional calculi and arrived at the following relative strength diagram (where 'arrow' as well as 'is in the same box' in the diagram means 'p-simulates'):

p-simulation chart

The main results are:

i. some system in one of the boxes is polynomially bounded iff all systems in the box are (for a fixed set of connectives);

ii. resolution and truth-tables are not polynomially bounded.

## References


[1] S.A. Cook and R.A. Rechkow, On the lengths of proof in the propositional calculus, Preliminary version. *Proc. Sixth Annual ACM Symposium on Theory of Computing* (1974), 135–148. Corrections for "On the lengths of proofs in the propositional calculus". SIGACT News (1974), 15–22.

[2] S.A. Cook and R.A. Rechkow, The relative efficiency of propositional proof systems. J. Symbolic Logic **44** (1979), 36–50.