Notes prepared by
Stanley Burris
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Comparing the expressive power of the propositional logic with the calculus of classes

The *universal* Aristotelian statements can be expressed by propositional formulas as follows:

universal statement	propositional logic translation
All S is P.	$S \implies P$.
No S is P.	$S \implies \neg P$.

Using this we have expressed the lengthy argument of Lewis Carroll in the propositional calculus *since all the statements are universal* in Example II.7.11 of LMCS.

Unfortunately we do not have a translation of I,O statements into propositional formulas. The simplest "upgrade" of the propositional calculus which is adequate to handle the I,O statements is the *monadic predicate calculus* which deals with quantified first-order statements about unary predicates.[1]

We can translate propositional logic into the Calculus of Classes by letting τ be the conversion of propositional formulas into Calculus of Classes terms obtained by simply replacing $\vee$ with $\cup$, $\wedge$ with $\cap$, and $\neg$ with $'$; and then observing that an argument

$$\begin{array}{c}\varphi_1\\ \vdots\\ \varphi_k\\ \hline \varphi\end{array}$$

is valid in the Propositional Logic iff

[1]Actually one can restrict oneself to the case that there is just a single first-order variable x available — such a logic is formulated in Hilbert and Ackermann and called the Calculus of Classes. However this is not the traditional formulation of the Calculus of Classes.

$$\begin{array}{c} \tau(\varphi_1) \approx 1 \\ \vdots \\ \tau(\varphi_k) \approx 1 \\ \hline \tau(\varphi) \approx 1 \end{array}$$

is valid in the Calculus of Classes.

Conversely, given an equational argument in the Calculus of Classes we can assume that it is in the form

$$\begin{array}{c} \varphi_1 \approx 1 \\ \vdots \\ \varphi_k \approx 1 \\ \hline \varphi \approx 1, \end{array}$$

and reversing our translation we have a corresponding argument in the propositional logic.

In summary we have a translation of arguments in the propositional calculus into arguments in equations in the calculus of classes, and conversely. Hence they can be thought of as equivalent. Both are adequate to handle the universal Aristotelian statements. To strengthen the Calculus of Classes to handle I and O statements we only need to add $\not\approx$. No such easy strengthening is available for the propositional logic.