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Rigid Boolean powers

STANLEY BURRIS[1]

Shelah proved that in the variety of Boolean algebras there are rigid algebras of every uncountable cardinality (see [2]). We will prove that certain rigid simple algebras S transfer this result to the variety generated by S.

If A is an algebra let $\text{Con}(A)$ denote the *lattice of congruences* of A, and let $\text{Aut}(A)$ be the *automorphism group* of A. For X a Boolean space let $A[X]^*$ be the algebra of continuous functions from X to A, giving A the discrete topology (this construction is called a *bounded Boolean power*). For $f, g \in A[X]^*$ let $[\![f = g]\!] = \{x \in X \mid fx = gx\}$. X^* is the *Boolean algebra of clopen subsets* of X.

LEMMA. *For any algebra A and Boolean space*[2] *X the map from* $(\text{Aut}(A))[X]^*$ *to* $\text{Aut}(A[X]^*)$ *given by* $\alpha \to \bar{\alpha}$ *where* $(\bar{\alpha}f)(x) = (ax)(fx)$, $f \in A[X]^*$, *is a group embedding. If this embedding is surjective then* X^* *must be rigid or* $|A| = 1$.

Proof. The first part is straight-forward as the mapping is defined componentwise. For the second part note that if $\mu : X \to X$ is a homeomorphism then the map from $A[X]^*$ to $A[X]^*$ defined by $f \to f \circ \mu$ is an automorphism of $A[X]^*$. If this automorphism is equal to $\bar{\alpha}$ for some $\alpha \in \text{Aut}(A)[X]^*$ then an easy argument shows $\bar{\alpha}$ is the identity map on the constant functions in $A[X]^*$, hence $\bar{\alpha}$ is the identity map on $A[X]^*$, so μ is the identity map on X or $|A| = 1$. Thus the embedding $\alpha \to \bar{\alpha}$ is surjective implies X^* is rigid, or $|A| = 1$.

It is trivial to show that if X^* is rigid then the map in the above lemma need not be surjective (let A be the group Z_2 and let X^* be an infinite rigid Boolean algebra). In the following we give sufficient conditions on A which ensure that the mapping is surjective, provided X^* is rigid.

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[2] This lemma actually holds for an arbitrary topological space X, where $A[X]^*$ is, as before, the algebra of continuous functions.

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University of Manitoba,
Winnipeg, Manitoba,
Canada.