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A note on directly indecomposable algebras

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Given an algebra **A** and a congruence θ of **A** let $\boldsymbol{\theta}$ be the subalgebra of $\mathbf{A}\times\mathbf{A}$ with universe θ. The results in this note depend on the following simple observation: if **A** belongs to a congruence distributive variety then the congruences of $\boldsymbol{\theta}$ are just the restrictions to θ of the congruences $\theta_1\times\theta_2$ of $\mathbf{A}\times\mathbf{A}$.

THEOREM 1. *If V is a congruence distributive variety such that every directly indecomposable member is subdirectly irreducible then V is semi-simple.*

Proof. Suppose **A** is a subdirectly irreducible non-simple member of *V*. Let θ be the unique atom of Con **A**. Then $\boldsymbol{\theta}$ is directly indecomposable but it is not subdirectly irreducible. □

COROLLARY 2. *A congruence distributive variety is Boolean representable* (*see Clark and Krauss* [1]) *iff it is a discriminator variety.*

Proof. If a congruence distributive variety is Boolean representable then it must be semi-simple by Theorem 1. The rest is in [1]. □

THEOREM 3. *A finitely generated congruence distributive variety is directly representable by finitely many finite algebras* (*see McKenzie* [2]) *iff it is semi-simple arithmetical.*

Proof. ($\Leftarrow$) This is well-known. ($\Rightarrow$) If it is directly representable then it has only finitely many finite directly indecomposable members. Suppose **A** is a finite non-simple directly indecomposable member of the variety. Let θ be a maximal congruence of **A**. Then $\boldsymbol{\theta}$ is directly indecomposable and $|\theta|>|A|$; repeating this

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leads to infinitely many finite directly indecomposable members, which is impossible. Thus each directly indecomposable member is simple, hence the free algebra on three generators is isomorphic to a product of finitely many finite simples. From this we conclude that the variety is congruence permutable. □

References

[1] PETER H. KRAUSS and DAVID M. CLARK, *Global subdirect products.* Memoirs of the AMS *17* No. 210 (1979).

[2] R. MCKENZIE, *Para primal varieties: A study of finite axiomatizability and definable principle congruences in locally finite varieties.* Alg. Universalis *8* (1978), 336–348.

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