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Bounded Boolean powers and $\equiv_n$

STANLEY BURRIS[1]

Sabbagh [2] has proved that two R-modules are elementarily equivalent whenever they satisfy the same first-order sentences which, in prenex normal form, have no more than two alternations of quantifiers. In particular this holds for Abelian groups. The situation is, on the other hand, just the opposite for Boolean algebras: Waszkiewicz [3] has shown that for any natural number n there are Boolean algebras B and C which are not elementarily equivalent but which nevertheless satisfy the same first-order sentences (in prenex normal form) which have no more than n alternations of quantifiers (we use the notation $B \equiv_n C$).

The following discussion shows, as an application of the results on bounded Boolean powers in Burris [1], that any variety which contains a finite B-separating algebra behaves just as the variety of Boolean algebras with respect to this point.

We use the notation A[B]* for bounded Boolean powers (as in [1]) and define A to be *B-separating* if $A[B_1]^* \cong A[B_2]^*$ implies $B_1 \cong B_2$.

THEOREM. *For any algebra A of type τ and Boolean algebras B_1, B_2, if $B_1 \equiv_n B_2$ then $A[B_1]^* \equiv_n A[B_2]^*$. If, in addition, A is finite and B-separating, then $B_1 \not\equiv B_2$ implies $A[B_1]^* \not\equiv A[B_2]^*$.*

Proof. The first assertion follows from the Feferman-Vaught Theorem for bounded Boolean powers (see [1], Lemma 4.1), and the second is stated in [1] as Theorem 4.3(x).

Hence, for example, if S_3 is the symmetric group on 3 letters and $n < \omega$ then there are groups G_1, G_2 in the variety generated by S_3 such that $G_1 \equiv_n G_2$ but $G_1 \not\equiv G_2$. We can make a similar assertion for every congruence distributive variety containing a non-trivial finite algebra (and hence a finite B-separating algebra).

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