Preface

In the late 1980s Kevin Compton published papers showing how to apply an analysis of the popular partition identities to obtain monadic second-order limit laws, including 0–1 laws, for numerous classes of graphs, posets, etc. I was fascinated by this work. When he gave a colloquium talk in the early 1990s in Waterloo on his recent work on a limit law for Abelian groups, the temptation to learn more about this subject was irresistible. This engaged a favorite area of my own research, algebraic structures.

Compton's papers can be somewhat opaque for specialists in combinatorics and number theory, as well as for specialists in logic, because of the intimate way he has woven these subjects together. After seeing the books [**29**], [**30**] of John Knopfmacher on abstract analytic number theory, it seemed worthwhile to separate Compton's treatment into two parts, one on density in number systems, and the other on the application of number theoretic density results to obtain logical limit laws. Each of these parts is of interest in its own right.

The reader will find a leisurely and detailed exposition of Compton's investigations, and closely related work of others, including recent contributions of Jason Bell, Edward Bender, Peter Cameron, Paweł Idziak, Arnold Knopfmacher, John Knopfmacher, Andrew Odlyzko, Bruce Richmond, András Sárközy, Cameron Stewart, Richard Warlimont, Alan Woods, and the author. The presentation is from the perspective of abstract number systems, in the spirit of John Knopfmacher's work in abstract analytic number theory.

This book has been used as an undergraduate special topics reading text at the University of Waterloo. Part 1, on Additive Number Systems, is completely accessible to an advanced undergraduate student. All chapters preceding Chapter 6 are devoted to number theoretic density, requiring only the usual undergraduate background in analysis, especially in power series, and an exposure to abstract mathematics. The well known ratio test plays a central role. The section on asymptotics, at the end of Chapter 5, uses basic complex analysis, including the Cauchy integral formula. Chapter 6 covers the logical aspects for Part 1. This chapter is self-contained so that one can work through it without prior exposure to logic. It features one of the most delightful tools of logic, the Ehrenfeucht-Fraissé games. (Having had a first course in logic, so that one is comfortable with first-order languages and structures, will no doubt make the chapter more rapid reading.)

Part 2, on Multiplicative Number Systems, offers the challenge and reward of becoming reasonably comfortable with Dirichlet series. The parallels with Part 1 show Dirichlet series as a natural companion of power series. Having worked through Part 1, one will be able to predict many of the results to be proved—the local density results of Part 1 seem, as if by magic, to reappear as global density results in Part 2. There is surely some deep connection between power series and Dirichlet series that we have not yet understood. The last chapter introduces the reader to the Feferman-Vaught Theorem, a favorite tool to analyze direct products, and Skolem's analysis of first-order sentences about Boolean algebras.

The reader will find all the material needed to thoroughly understand the method of Compton for proving logical limit laws. Above all, I think one will be delighted to see so many interesting tools from elementary mathematics pull together to help answer the question "What is the probability that a randomly chosen structure has a given property?"

Thanks go to Paweł Idziak for his contributions to the study of limit laws when I was first starting to work in the area, to Andrew Odlyzko for helping me understand what was going on with the Dirichlet series, to András Sárközy for helping develop the general multiplicative theory of limit laws during a visit to Waterloo, to Cameron Stewart for discussions of the ratio test, to Dejan Delic for help with proofreading an early draft of the book, to Bruce Richmond for helping me to locate and understand several relevant results from asymptotics, to John Knopfmacher for challenging me to take a harder look at what was going on with logical limit laws, to Richard Warlimont for a remarkable amount of beautifully handwritten correspondence regarding ways to improve the presentation, to Jason Bell for keeping me informed of his recent research in this area, and to Karen Yeats, a second year mathematics undergraduate at U. Waterloo who eagerly read the entire manuscript during the summer of 2000, giving me detailed feedback on how the text comes across to an undergraduate. Kevin Compton is, of course, the ultimate inspiration for this work, through his publications and his elegant lectures over the years.

Edward Dunne, Christine Thivierge and Elaine Becker of the AMS Book Program did everything possible to make the transition from manuscript to book a pleasant and trouble-free experience; and Barbara Beeton of the AMS Technical Support group was (as always!) able to solve all of my Tex related problems with the document.

Finally, I want to thank the Natural Sciences and Engineering Research Council of Canada for their long standing support of my investigations in universal algebra, logic, and computation, the support that has made it possible to write this book.

For errata and updates see www.thoralf.uwaterloo.ca.

Stanley Burris
Waterloo, 2000