Method of Undetermined Coefficients

A method for finding a particular solution, $y_p(x)$ of a constant coefficient, inhomogeneous, second order DE

$$\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = r(x) \tag{1}$$

where $b, c \in \mathbb{R}$ and $r(x)$ consists of products or sums of sines, cosines, exponentials and nonnegative integer powers of x.

Idea: Use the form of $r(x)$ to determine a form of $y_p(x)$.

Procedure:

1. Given $r(x)$ choose the form for $y_p(x)$ according to the following table.

Term in $r(x)$	Form of $y_p(x)$
$e^{\gamma x}$	$Le^{\gamma x}$
x^n	$K_n x^n + K_{n-1}x^{n-1} + \cdots + K_1 x + K_0$
$\cos \omega x$	$M_1 \cos \omega x + M_2 \sin \omega x$
$\sin \omega x$	

   K_j, M_j, L are unknown constants which will be determined.

2. If $r(x)$ is a product of terms in column 1, choose $y_p(x)$ to be the product of the corresponding terms in column 2.

3. If your choice for $y_p(x)$ is a solution of the associated homogeneous equation, then multiply it by the lowest power of x so that it is no longer a solution.

4. If $r(x)$ is a sum of functions

$$r(x) = r_1(x) + r_2(x) + \cdots + r_m(x)$$

choose an appropriate $y_{p_j}(x)$ for each $r_j(x)$, following steps 1–3, and then add them together:

$$y_p(x) = y_{p_1}(x) + y_{p_2}(x) + \cdots + y_{p_m}(x).$$

5. Substitute the form you have found for $y_p(x)$ into the nonhomogeneous equation (1) and determine the values for the unknown coefficients so that $y_p(x)$ satisfies the equation.