Qualitative Method of Analysis for First Order Equations

1. Start with equation standard form: $\dfrac{dy}{dx} = f(y)$.

2. Identify equilibrium solutions by looking values of y such that $f(y) = 0$. Determine where $\dfrac{dy}{dx}$ is positive and negative.

3. Calculate $\dfrac{d^2y}{dx^2} = f(y)f'(y)$. Determine where $\dfrac{d^2y}{dx^2}$ is positive, negative and zero.

4. Make the correspondence table as below and find the qualitative description of $y(x)$.

Interval in y	Description $f(y)$	Sign of $\dfrac{dy}{dx} = f(y)$	Sign of $\dfrac{d^2y}{dx^2} = f(y)f'(y)$	Description of $y(x)$
	positive increasing	$+$	$+$	increasing concave up
	positive decreasing	$+$	$-$	increasing concave down
	negative increasing	$-$	$-$	decreasing concave down
	negative decreasing	$-$	$+$	decreasing concave up

5. Sketch the solutions $y(x)$ vs x for various initial conditions $y(0) = y_0$.

6. Classify equilibrium solutions as stable or unstable.