

1. For the vector DE's given in question 2 b)i),ii),iv) on p. 156 of the Course Notes find the special fundamental matrix, $\Phi(t)$, using the method discussed in class. Note: on Assignment 6, you solved some of these using the the eigenvalue method. You may use those results in your answer.
2. Show that the fundamental matrices you found in 1 ii) and iv) satisfy the following properties:

$$\Phi(0) = I, \quad [\Phi(t)]^{-1} = \Phi(-t).$$

3. Solve the initial value problem

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x}, \quad \mathbf{x}(0) = \mathbf{x}_0,$$

where A and \mathbf{x}_0 are given below, using the fundamental matrices you found in the last question.

$$\begin{aligned} \text{(i)} \quad \mathbf{A} &= \begin{bmatrix} -3 & 2 \\ 2 & -3 \end{bmatrix}, \quad \mathbf{x}_0 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \\ \text{(ii)} \quad \mathbf{A} &= \begin{bmatrix} -1 & -4 \\ 1 & -1 \end{bmatrix}, \quad \mathbf{x}_0 = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \end{aligned}$$

4. Problem 5 b) (i),(ii) on page 157 of the Course Notes. You may use the fundamental matrices you found in problem 1 to answer this question.
5. Problem 6 a) b) c) d). Note that c) and d) refer to the homogeneous equation.