

AM 250**Assignment 4****Due Wednesday, February 1**

Course Notes references: pp. 29–30 (Electric Circuits); pp.33–48 (Dimensional Analysis)

1. An RC circuit consists of a 20 ohm resistor, a 10^{-2} farad capacitor and a battery supplying a voltage given by $20te^{-5t}$ volts. If there is no initial charge on the capacitor, find the steady-state charge, the transient charge and the maximum charge. Plot the charge as a function of t .
2. A 10 ohm resistor and a 5 henry inductor are connected in an electric circuit with an applied voltage given by $10t$ volts. Find the current as a function of time if there is initially no current in the circuit. Find the steady-state and transient current.
3. Course Notes Problem Set 2 # 2. (Note: $\sin \omega t$ is dimensionless).
4. Course Notes Problem Set 2 # 4 (a),(b),(d). (See p. 38 of the Course Notes).
5. The population of mushrooms in a field grows at a rate that is proportional to the number present. The mushrooms are picked at a constant rate H per day and initially there are N_0 mushrooms present. A reasonable model for this situation is

$$\frac{dN}{dt} = rN - H, \quad N(0) = N_0$$

where N is the number of mushrooms in the field, t is time and r is a constant.

- (a) If t is measured in days, what will be the units of r ?
- (b) Find dimensionless variables for the model and rewrite the model in terms of these new variables.
- (c) Use qualitative analysis to sketch the solutions of the model in dimensionless form.
- (d) Discuss the longterm ($t \rightarrow \infty$) behaviour of solutions and what this means for the population.

6. (Archimedes Principle) The buoyant force, B , is what causes an object to float in certain fluids. Experimentally, one can determine that this force depends on the volume V and mass m of the object, the density ρ of the fluid in which it is immersed and the gravitational acceleration g . Use dimensional analysis to find (as accurately as possible) how B depends on V, m, ρ and g .
7. Suppose that the moon moves about the earth on a (roughly) circular path with period T . Assuming that T depends on the mass of the earth, M , the mass of the moon, m , the distance between the centres of mass of the two objects, r , and the universal gravitational constant G , derive an expression for T using dimensional analysis.