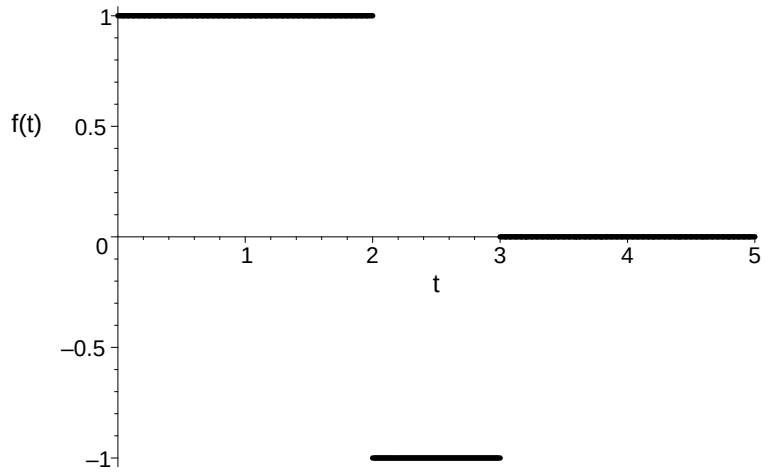
AMATH 250 Assignment 10 Due Wednesday March 29

Course Notes References: Sections 4.1.5, 4.2.3, 5.3

1. Consider the function $f(t)$ having the graph below.



   (a) Use the definition to find the Laplace Transform of the function.

   (b) Find an expression for the function using step functions and calculate the Laplace Transform using this expression and known transforms.

2. Sketch the following function, then find its Laplace Transform.
   $f(t) = (t-1)H(t-1) - 2(t-2)H(t-2) + (t-3)H(t-3)$.

3. Find the Inverse Laplace Transform of each of the following functions.

   (a) $F(s) = \dfrac{e^{-\pi s}}{s^2+1}$

   (b) $F(s) = \dfrac{e^{-2s}}{s^2+s-2}$

4. Solve the following initial value problem using Laplace Transforms

$$y'' + 4y = H(t-2\pi)\sin(t),\ y(0) = y'(0) = 0.$$

5. A circuit has inductance 1 henry, resistance 5 ohms and capacitance 1/6 farad. It is connected to a generator which can supply a constant voltage of 2 volts. There is no charge or current in the circuit when the generator is turned on. If it is turned on for one second and then turned off, an appropriate model for the system is:

$$Q'' + 5Q' + 6Q = 2(1 - H(t-1));\ Q(0) = 0,\ Q'(0) = 0$$

where $Q(t)$ is the charge in the circuit at time t.

(a) Justify the model given above.

(b) Use Laplace Transforms to find the charge at any time $t \geq 0$.

6. Problem 14 on p. 154 of the Course Notes.

7. Use Laplace Transforms to solve the vector IVP

$$\mathbf{x}' = \mathbf{A}\mathbf{x},\ \mathbf{x}(0) = \mathbf{x}_0$$

where

$$\mathbf{A} = \begin{bmatrix} -1 & -4 \\ 1 & -1 \end{bmatrix} \quad \text{and} \quad \mathbf{x}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$