Pairwise comparisons of treatment groups via Eulerian tours and Hamiltonian decompositions

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We present improved graphical displays for two classical data analysis problems. Conventionally in one-way anova, treatment differences are visualized by drawing confidence intervals for all pairs of treatments. Hsu and Peruggia [2] and more recently Heiberger and Holland [1] devised a rather visually complicated display which also presents the group means along with the pairwise comparisons. Our display consists of horizontal boxplots for each treatment where boxplots are repeated as necessary so that each pair of treatments appear adjacently for ease of comparison. To enhance the comparison, a (Tukey HSD) confidence interval is drawn between every pair of treatments, so that significantly different treatments are easily identified. Furthermore, the confidence intervals may be ordered to give prominence to important comparisons.

In two-factor anova, treatment interactions are visualized by constructing an interaction plot, where near-parallelism of line segments suggests non-significant treatment interactions. We present examples where assessment of parallelism is affected by the order of factor levels. As a remedy, we advocate the dependence on order by repeating treatments so the interaction plots show line segments connecting all pairs of treatment levels for one factor while holding the second factor fixed.

In both of the above problems, the improved graphical displays make use of sequences of treatment groups where groups are repeated so that all groups appear adjacently. These sequences amount to finding eulerian paths on the complete graph where each vertex represents a treatment group. We present a variety of algorithms for constructing eulerian paths, for the situations where the graph edges are weighted, and where a hamiltonian decomposition is required. Other applications of eulerian tours and hamiltonian in statistical graphics include star glyphs and parallel coordinates.

References

[1] Heiberger, R.M., and P. Holland. (2006), “Meanmean multiple comparison displays for families of linear contrasts.”. *Journal of Computational and Graphical Statistics*, 15, 937955.

[2] Hsu, J. and M. Peruggia (1994), “Graphical representation of Tukey’s Multiple Comparison Method”, *Journal of Computational and Graphical Statistics*, 3, 143-161.