

# Interactive Clustering

## Overview and Tools

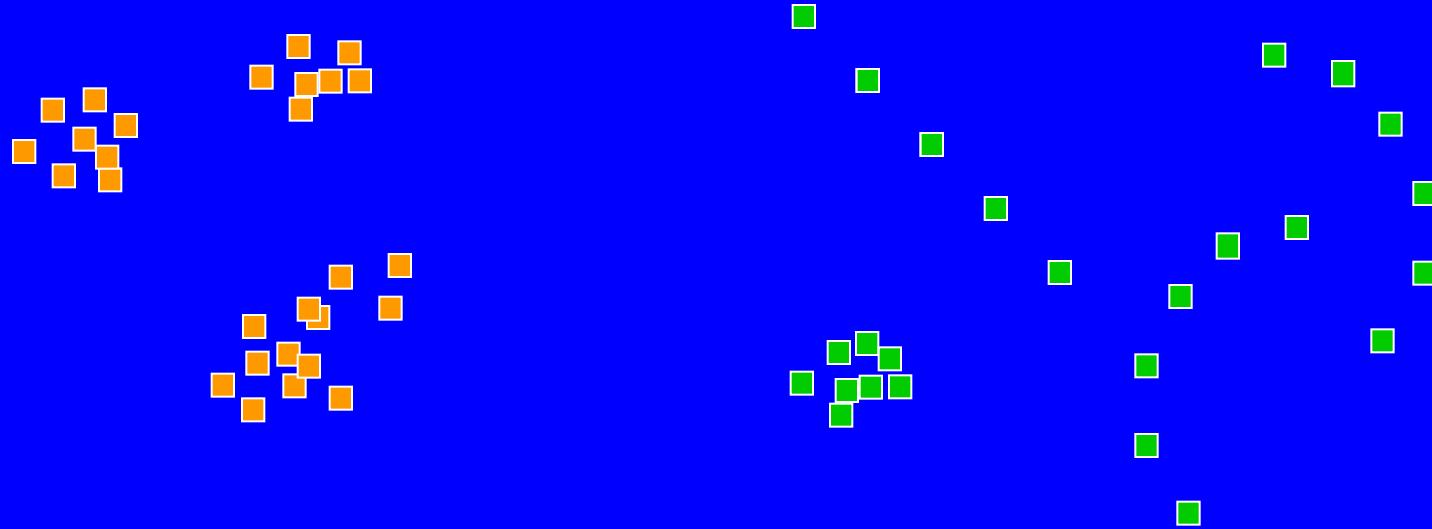
Wayne Oldford  
University of Waterloo

# Overview

1. Finding groups in data
2. Interactive data analysis
3. Enlarging the problem
4. Putting it together
5. Software modelling  
(illustration)
6. Summary

# 1. Finding groups in data

- Objects to be grouped together
  - locations
  - pairwise (dis)similarities
- **Applications:**
  - Web documents as objects to be grouped
  - Building groups to use later as classification
  - Building groups to serve as templates
  - Building groups to understand/model



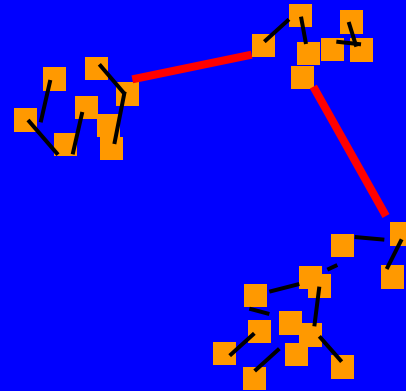
Group definition (like with like)

- homogeneous vs heterogeneous
- part of pattern

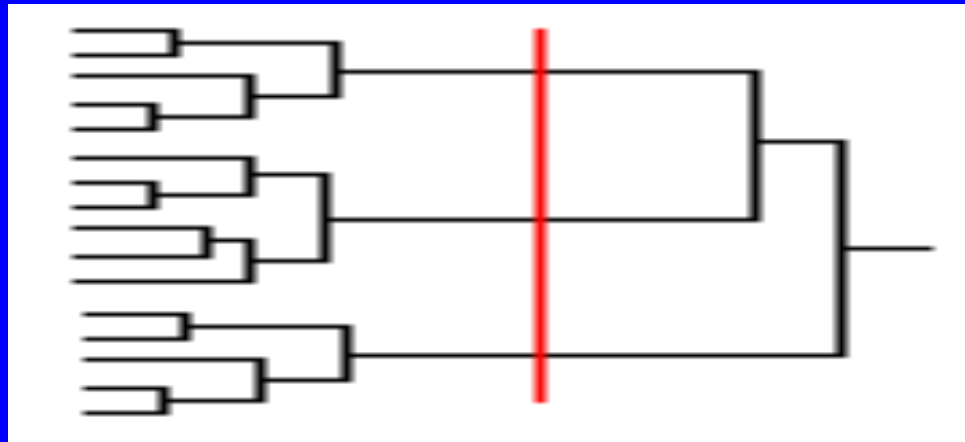
group definition is a problem

# Clustering approaches

- Agglomerative (near points/clusters are joined)
  - Single linkage
  - Complete linkage
  - Average linkage
- Recursive splitting
  - e.g. minimal spanning tree



# Cluster hierarchies

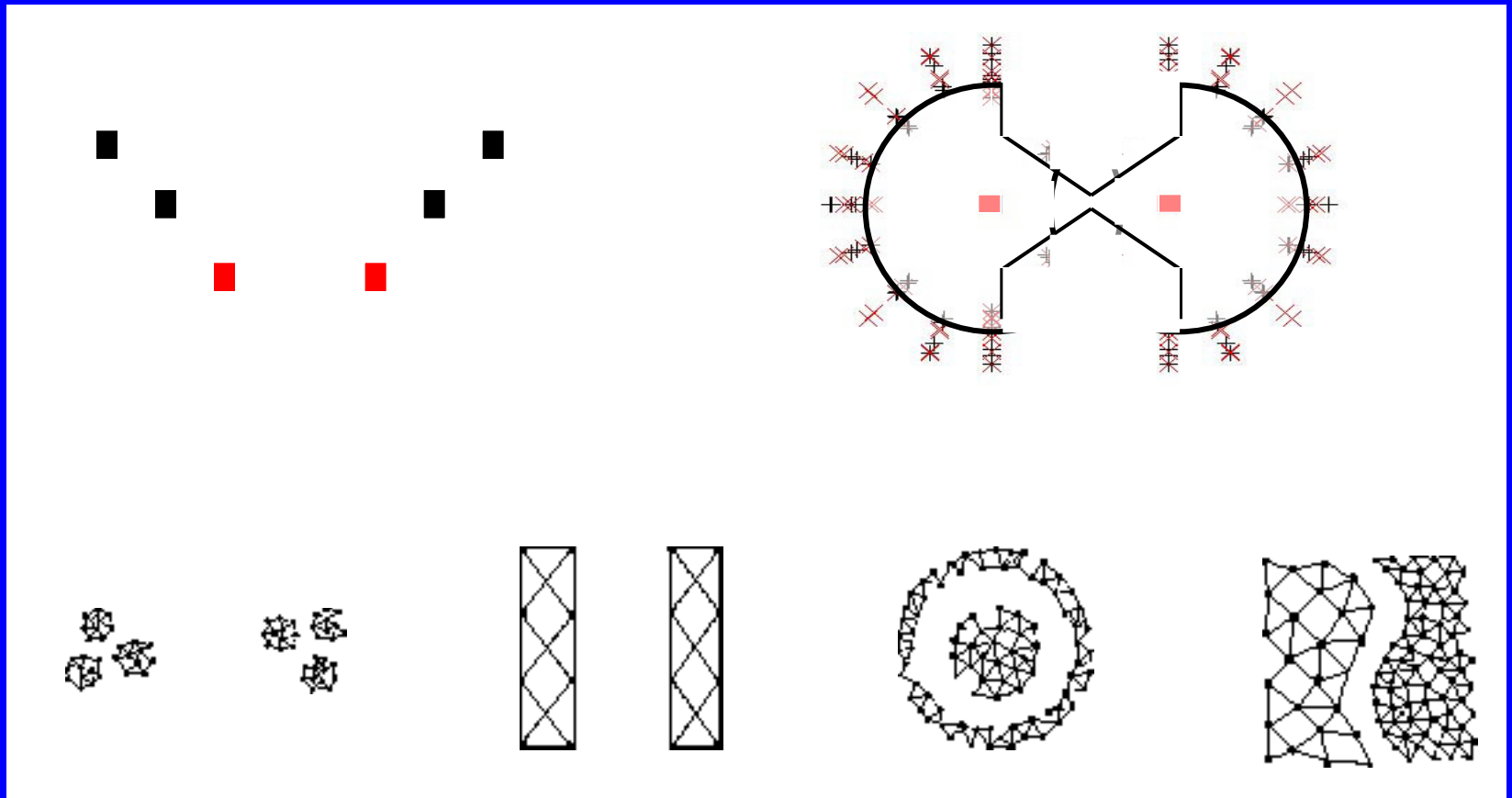


- Clusters are nested
- Often represented as a tree (dendrogram)
- Join/split history and ‘strength’ preserved

# Other approaches

- k-means
  - assign points to k groups
  - re-assign to improve objective function
- model-based
  - likelihood/Bayesian; model search/averaging
- density estimation
  - groups = high-density regions
- classification to cluster
- visually motivated methods

# Visual Empirical Regions of Influence (VERI)





# Notes

- many choices
  - between and within methods
- built-in biases for shapes
- computationally costly
  - $O(n^2)$  ...

Conceptual model: algorithmic, run to completion

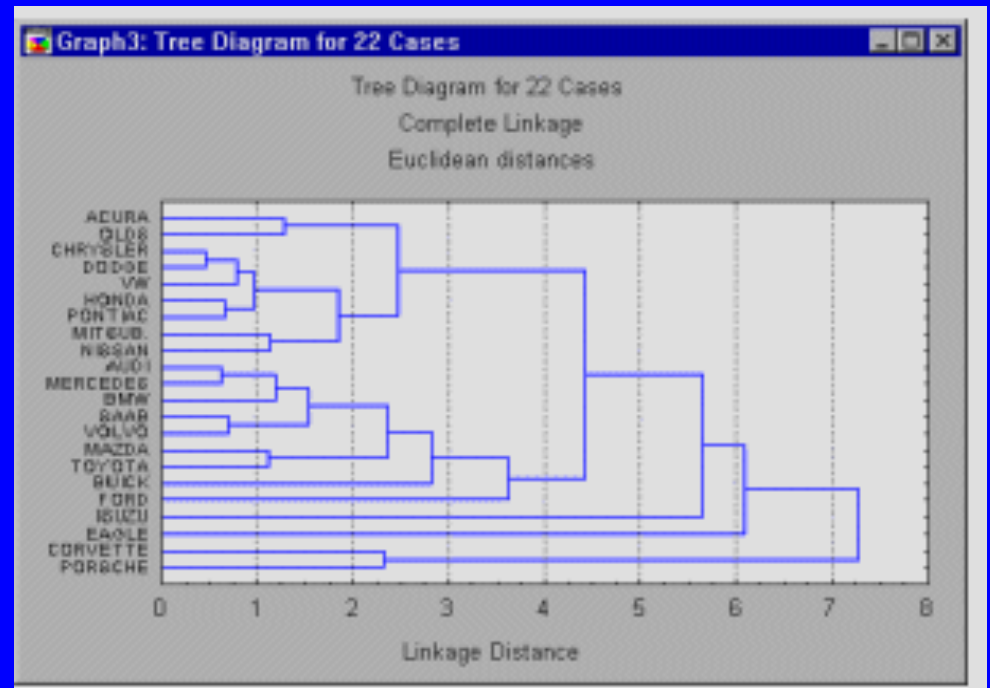
# typical software

- resources dedicated to numerical computation

- teletype interaction

- runs to completion

- graphical “output”



## Compare to interactive data analysis

May 17 2001

CASI 2001

# Interactive data analysis

- exploratory, tentative
- graphical
- non-algorithmic
  - varied granularity
- integrated
- deep interaction

### 3. Enlarging the problem

Mutually exclusive and exhaustive groups

$$g_1, g_2, \dots, g_k$$

form a partition

$$P = \{g_1, g_2, \dots, g_k\}$$

of the set of data objects.

Goal: Explore the space of possible partitions.

# Structuring the partition space

$$P_A = \{g_1, g_2, \dots, g_a\} \text{ and } P_B = \{h_1, h_2, \dots, h_b\}$$

- When  $a > b$ ,  $P_A$  call a *finer partition* than  $P_B$ .
  - $P_A$  is called a *refinement* of  $P_B$  (or  $P_B$  a *reduction* of  $P_A$ )
- $P_A$  is *nested* in  $P_B$  only if  $a > b$  and every  $g_i$  is a subset of a single  $h_j$  - write  $P_A \} P_B$  or  $P_B \{ P_A$
- When  $a = b$ ,  $P_A$  is called a *reassignment* of  $P_B$

# Reduction

$P_1 = \{g_1, \dots, g_6\} \rightarrow P_2 = \{h_1, \dots, h_4\} \rightarrow P_3 = \{m_1, m_2, m_3\}$

- $h_i = g_i \quad i = 1, 2$  ;  $h_3 = \text{join}(g_3, g_4)$  ;  $h_4 = \text{join}(g_5, g_6)$

– nesting:  $P_1 \} P_2$

- **disperse** elements of  $h_4$  over  $h_i \quad i = 1, 2, 3$  to give  $m_i$  for  $i = 1, 2, 3$ .

–  $\text{split}(h_4) = \{h_1^*, h_2^*, h_3^*\}$  ;  $m_i = \text{join}(h_i^*, h_i)$

–  $P_2 \} P_3$  is false

# Reduction decisions/options

- **join** operations: which groups?
  - e.g. inner, outer, centres, ...
  - distance measures to use ...
- **dispersal** operations:
  - selecting group(s)
    - Max volume, eigen-value, MST...
  - determining **partitional** method
    - random, VERI, MST, ...
  - choosing **join** ...



# Refinement

$$P_2 = \{h_1, \dots, h_4\} \quad \dashrightarrow \quad P_1 = \{g_1, \dots, g_6\}$$

- $g_i = h_i \quad i = 1, 2$  ; **split** ( $h_3$ )  $\rightarrow$   $g_3, g_4$   
**split** ( $h_4$ )  $\rightarrow$   $g_5, g_6$

nesting:  $P_2 \{ P_1$

# Refinement decisions/options

- **which groups** to split?
  - e.g. inner, outer, directions, ...
  - distance measures to use ...
- **how** to split?
  - MST, outlying points, reassignment, ...

# Reassignment

$$P_1 = \{g_1, \dots, g_k\} \rightarrow P_2 = \{h_1, \dots, h_k\}$$

- objective function  $d(P)$  to be minimized.  $P \leftarrow P_1$
- for each object  $o$  in  $g_i$ , assign it to one of  $g_j$  ( $j \neq i$ ) forming a new partition  $P_{ij}$  and find largest  
$$\Delta_{ij}(o) = d(P) - d(P_{ij})$$
- repeat for all  $i, j$ . If  $\max \Delta_{ij} > 0$  move  $o$  from  $g_i$ , to  $g_j$
- Repeat until  $\Delta_{\max} \leq 0$

# Reassignment decisions/options

- **Objective function**
  - distances, centres, ...
  - within vs between/within, ...
  - variates/directions
- **Iteration strategy**
  - single-pass, k-means, complete looping (greedy), start, ...

# 4. Putting it together

Series of moves in partition space:

1. **Refine** (P)  $\rightarrow P_{\text{new}}$

2. **Reduce** (P)  $\rightarrow P_{\text{new}}$

3. **Reassign** (P)  $\rightarrow P_{\text{new}}$

# Additional ops on partitions

- Unary:
  - **Subset** (P)
  - Operate any of **R** (subset (P))
  - **Manual** (P) ... change P according to manual intervention (e.g. colouring)

# n-ary operators

- **resolve**  $(P_1, \dots, P_m) \rightarrow P_{\text{new}}$
- **dissimilarity**  $(P_i, P_j) \rightarrow d_{i,j}$
- **display**  $(P_1, \dots, P_m)$ 
  - **dendrogram** if  $P_1 \{ \dots \{ P_m$
  - mds plot of all **clusters** in  $P_1, \dots, P_m$
  - mds plot of all **partitions**  $P_1, \dots, P_m$

# 5. Software modelling

- Principal control panel:
  - current partition and list of saved partitions
  - refine, reduce, re-assign, re-start buttons
  - cluster plot button (mds plot)
  - random select button
  - subset focus and join toggle
  - operation on partitions button
  - manual button (form partition from point colours)



# Secondary panels

- Refine:
  - performs refine, offers access to arguments
- Reduce
  - performs reduce, offers access to arguments
- Reassign
  - performs reassign, offers access to arguments
- Each will operate on only those points **highlighted** or **on all** if none selected.

# Secondary panels (continued)

- Operate on partitions
  - saved partitions list
  - resolve selected partition
  - plot selected partitions using selected dissimilarity
  - dendrogram of selected partitions (if nested)
  - cluster-plot for clusters of selected partitions (esp. for non-nested)

# Software modelling (details)

- Objects:
  - Point-symbols, case-objects (existing in Quail)
  - Cluster-points
  - Clusters
  - Partitions
- Methods
  - Reduce, refine, reassign, ...

# Software illustration

- Two prototype displays (buggy)
  - Single-window
  - Separate windows
- Integration with existing Quail graphics
- Manual, dendrogram, cluster plots, ...
- VERI clustering

# 6. Summary

- Cluster analysis is naturally exploratory and needs integration with modern interactive data analysis.
- Enlarging the problem to partitions:
  - simplifies and gives structure
  - encourages exploratory approach
  - integrates naturally
  - introduces new possibilities (analysis and research)

# Acknowledgements

- Erin McLeish, several undergraduates and graduate students in statistical computing course at Waterloo
- Quail: Quantitative Analysis in Lisp  
<http://www.stats.uwaterloo.ca/Quail>