PMath 911 Topics in Logic: Stability Theory **Assignment # 4**
Winter 2013, Rahim Moosa

Due on Tuesday, April 2nd

Except in question 5, we work throughout in a sufficiently saturated model $\mathcal{U}$ of a complete totally transcendental theory, T.

1. Submit today (Thursday March 21) your outline for the term project.

2. Suppose X is an M-definable where M is a model. Then there are exactly $\mathrm{dM}(X)$-many generic types of X over M. (We do *not* assume $\mathcal{M}$ is $\aleph_0$-saturated.)

3. Show that for all tuples a and b and all sets $A \subseteq B$, $ab \mathop{\smile\!\!\!\!\!\!|}_{A} B$ if and only if $a \mathop{\smile\!\!\!\!\!\!|}_{A} B$ and $b \mathop{\smile\!\!\!\!\!\!|}_{Aa} B$.

4. Suppose M is a model, $(G, \cdot)$ is an M-definable group, and S is the set of generic types in G over M. Show that Lemma 10.12 defines a group action of G on S.

5. Prove that finite extensions of definable fields are definable. To be precise: suppose L is an arbitrary language and $\mathcal{M}$ an arbitrary L-structure. Suppose $(F, 0, 1, +, -, \times)$ is a definable field in $\mathcal{M}$ and K is an abstract finite algebraic extension field of F. Prove that there exists a definable field extension of F in $\mathcal{M}$ that is isomorphic as a field to K, under an isomorphism that is the identity on F.

6. Suppose $T = \mathrm{DCF}_0$ and $k \subset U$ is a δ-subfield. You may use the following fact from differntial algebra: if $I \subsetneq J$ are prime δ-ideals in $k\{X\}$, then there is $f \in J$ with $\mathrm{ord}(f) < \mathrm{ord}(g)$ for any $g \in I$.

(a) Let $p(x) \in S_1(k)$ be such that I_p, the prime δ-ideal corresponding to p, is nontrivial. Prove that $\mathrm{RM}(p) \leq \min\{\mathrm{ord}(f) : 0 \neq f \in I_p\}$.

(b) Suppose $p \in S_1(k)$ is the type of a differentially transcendental element. Prove that $\mathrm{RM}(p) = \omega$.