PMath 911 Topics in Logic: Stability Theory **Assignment # 3**
Winter 2013, Rahim Moosa

Due on Tuesday March 12th

1. Submit by **Thursday March 7th**, as a separate document, a proposal for your Term Project. See the information sheet for details.

2. Suppose (X, cl) is a matroid, $A \subseteq X$, and $a_1, \ldots, a_n \in X$. Show that $\{a_1, \ldots, a_n\}$ is cl-independent over A if and only if for every $i = 1, \ldots, n$, $a_i \notin \mathrm{cl}(Aa_1 \ldots a_{i-1})$.

3. Prove that in a strongly minimal structure $\mathcal{M}$, for every $A \subseteq M$, there is a unique type $p \in S_n(A)$ with $\mathrm{RM}(p) = n$. This is called the *generic* n-type over A, and a realisation of this type is called a *generic n-tuple over A*.

4. In each of the basic examples of a strongly minimal theory, namely the theory of infinite sets, the theory of infinite F-vector spaces, and the theory of algebraically closed fields, compute the Morley rank of complete types and describe the generic types. Justify your answers.

5. (U-rank). Suppose T is a totally transcendental theory and $\mathcal{U}$ is a sufficiently saturated model. We let *U-rank* be the "foundation rank" for complete types, with repect to forking extensions. That is, we inductively define $U(p)$ for any $p \in S_n(A)$, by $U(p) \geq 0$ for all $p \in S_n(A)$, and $U(p) \geq \alpha + 1$ if there exists a forking extension $p \subset q \in S_n(B)$, $B \supset A$, with $U(q) \geq \alpha$. For limit ordinals we do the usual thing. And then we define $U(p) = \alpha$ in the natural way. Show that for all $p \in S_n(A)$,

(a) $U(p) < \infty$, i.e., U-rank is ordinal valued
(b) $U(p) = 0$ if and only if p is algebraic (has finitely many realisations).
(c) $U(p) \leq \mathrm{RM}(p)$

6. Let $L = \{P_0, P_1, \ldots\}$ be a language made up of $\aleph_0$-many unary predicate symbols, and suppose T is the theory saying

$\forall x\big(P_i(x) \to P_{i-1}(x)\big)$, for each $i > 0$,
$\neg P_0(x)$ has infinitely many realisations
$\big(P_i(x) \wedge \neg P_{i+1}(x)\big)$ has infinitely many realisations for all $i \geq 0$

Do the following:

(a) Give a breif sketch of why T admits QE and is totally transcendental.
(b) By QE there is a unique type $p(x) \in S_1(\emptyset)$, containing all the $P_i(x)$. Show that $\mathrm{RM}(p) = 2$ while $U(p) = 1$.