Quantum symmetries of quantum spaces

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Purpose of the study:

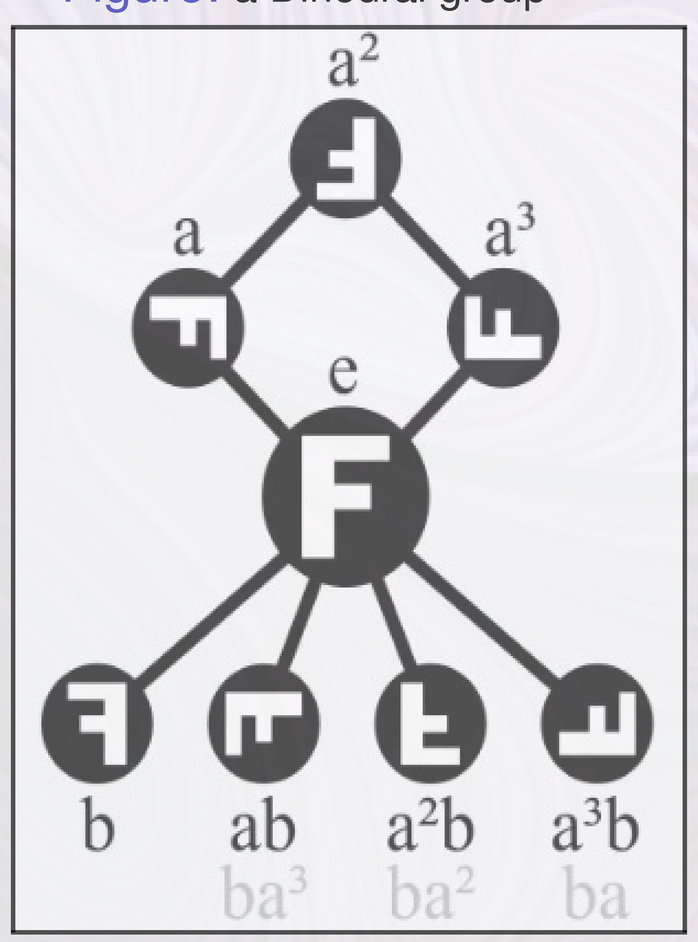
• Determine when a given **quantum space** admits a certain **quantum symmetry**, and conversely

•• does every quantum symmetry **act** on some quantum space?

Methods: What is *"a symmetry"*?

Groups encode the idea of symmetry. **A group** is just a multiplication table governing how its elements combine, and they arise in a multitude of ways:

Figure: a Dihedral group [Wi]



► In geometry:

The self-similarities of elementary shapes, as depicted to the left.

►► In physics:

Noether's Theorem relates the symmetries and conserved quantities of a physical system.

Illustrations of this result are the spatial-translation invariance in Newtonian mechanics corresponding to the conservation of linear momentum, and time-translation invariance paired with the conservation of energy.

Slogan: Groups are always accompanied by their *representations*.
Groups **act** on spaces.

Methods: What is *"a quantum symmetry"*?

The symmetries of objects arising from quantum physics cannot be framed within group theory. *Observables* of modern physics are not real numbers, where the order of measurement does not alter the outcome. Instead, they form a **von Neumann algebra** (vNA), whose elements *need not commute*; i.e.

$$a \cdot b \neq b \cdot a.$$

(Heisenberg's uncertainty principle at the experimental level.)

These vNA's are particular examples of so-called **C*-algebras**, and the study of their symmetries requires the language of tensor categories. **Tensor categories** [EGNO] simultaneously generalize groups and their representations, whose morphisms often admit graphical representations such as:

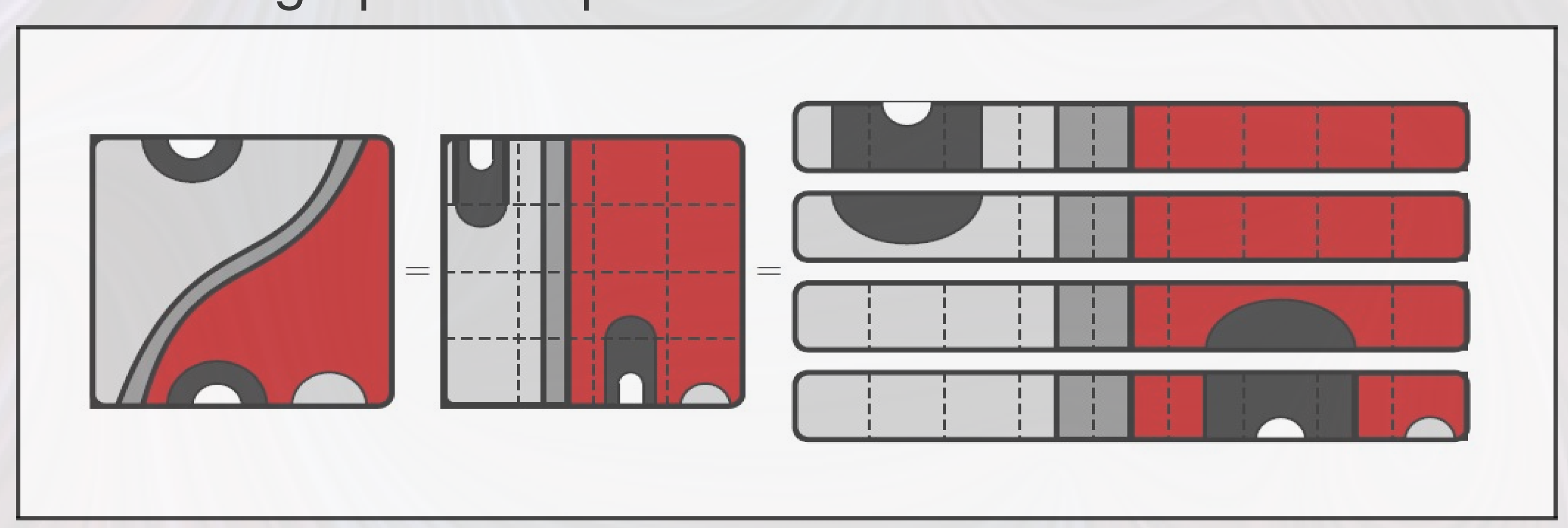


Figure: diagrams in TLJ, *the universal tensor category*

We therefore define a **quantum symmetry as a tensor category**.

Methods: What is *"a quantum space"*?

By Gelfand's Theorem in **Operator algebras**, we can trade a commutative C*-algebra for some *classical space*. Broader interpretation of this result bears the conclusion that *a general (i.e. non-commutative) C*-algebra remembers the shape of some* **quantum space**.

Maxim: Quantum spaces are C*-algebras.

Findings: *"The shape of quantum space"*

Theorem: Every tensor category of quantum symmetries acts on some (C*-)quantum space. [HaHe]

We have universal examples of these actions: In [FeHe], we **completely classified all *discrete quantum spaces*[DcY13], where the category is given by TLJ** in terms of weighted graphs.

Implications and predicted findings:

♣ We addressed our questions in the affirmative!

♣♣ This bridge between tensor categories and C*-algebras will allow for applications into the theory of operator algebras and dynamical systems.

♣♣♣ Tensor categories model particle excitations in condensed matter physics and can help understand *topological phases of matter.*(*)

♣♣♣♣ The question whether any quantum symmetries act on classical classical spaces remains open.

References:

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[DcY13] De Commer and Yamashita, *Tannaka-Krein duality for compact quantum homogeneous spaces. I* arXiv:1211.6552v3
[EGNO] P. Etingof, S. Gelaki, D Nikshych, and V. Ostrik, *Tensor Categories*, Math. Surveys and Monographs, V. **205**, AMS, Providence, RI, 2015, ISBN:9781470420246
[FeHe19] G. Ferrer, RHP, *Classifying Module Categories for Generalized TLJ $*$-2-Categories*, arXiv:1905.00471
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Background image from:
https://kevinwalker.info/#hair8_2_v8330_s75383_detail/2/6
[Wi] Figure taken from https://en.wikipedia.org/wiki/Dihedral_group
(*): 2016 Nobel Prize in Physics was awarded to physicists who established the importance of topology in understanding certain forms of matter.