Supplementary Material for "Controlling individual and experimentwise error rates in replicated regular two-level factorial experiments"

Pengfei Li[a], Oludotun J. Akinlawon[b], and Shengli Zhao[c]

[a]Department of Statistics and Actuarial Science, University of Waterloo, Waterloo, ON, Canada; [b]Knowledge Utilization Studies Program, University of Alberta, Edmonton, AB, Canada; [c]School of Statistics, Qufu Normal University, Qufu, China

ARTICLE HISTORY
Compiled January 13, 2022

ABSTRACT
This is the supplementary material for the paper submitted to the *Communications in Statistics - Simulation and Computation*. Section 1 presents more simulation results. Section 2 includes the R function to implement the proposed methods.

KEYWORDS
Experimentwise error rate; Individual error rate; Jackknife method; Lenth's method

1. More simulation results

In this section, we present simulations to compare the performance of our methods for controlling the IER and EER with the existing methods for both location and dispersion models in 2^5 and 2^6 factorial experiments.

1.1. Additional Simulation Results for Location Model

Case I: σ_i^2 homogeneous

For the 2^5 experiment with five two-level factors, A, B, C, D, and E, we use the model

$$y_{ij} \sim N(10 + 0.25A + 0.25B + 0.25D + 0.2BD, 1).$$

For the 2^6 experiment with four two-level factors, A, B, C, D, E, and F, we use the model

$$y_{ij} \sim N(5 + 0.15A + 0.15B + 0.15D + 0.12BD, 1).$$

We test the significance of the factorial effects of interest for each model at the 5% level based on the above methods. The simulation is repeated $N = 20,000$ times for each

CONTACT Shengli Zhao. Email: zhaoshli758@126.com

model. We compute the percentage of rejection of the null hypothesis $H_0 : \alpha_l = 0$, $l = 1, \ldots, I$. Throughout this section, $I = 31$ and 63 for the 2^5 and 2^6 designs, respectively. The results are summarized in Tables 1 and 2. We present only the results for the main effects and two-factor interaction effects because the results for three-factor or higher-order interactions are similar to the results for the effects that are not in the model.

Table 1. Percentage of rejection of the null hypothesis $H_0 : \alpha_l = 0$ at the 5% level for model $y_{ij} \sim N(10 + 0.25A + 0.25B + 0.25D + 0.2BD, 1)$ in replicated 2^5 experiments.

Effect	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Our method				WH method				Lenth's method			
A	66.4	79.1	87.7	92.8	67.7	79.4	87.8	93.0	54.0	65.8	77.1	84.5
B	65.8	79.6	87.9	92.8	67.1	80.1	88.1	92.9	52.7	67.1	76.4	84.5
C	4.6	5.0	5.0	4.8	4.9	5.1	5.1	4.9	3.3	3.2	3.6	3.7
D	65.7	79.3	88.3	92.8	66.9	79.7	88.5	92.9	53.1	66.6	77.1	84.3
E	4.5	4.7	4.7	4.8	4.8	4.8	4.8	4.9	3.2	3.2	3.4	3.7
AB	4.7	4.6	4.6	4.7	5.1	4.8	4.8	4.8	3.3	3.3	3.6	3.7
AC	4.6	4.6	4.7	5.0	4.9	4.8	4.7	5.0	3.1	3.2	3.4	3.7
BC	4.7	4.8	4.8	4.8	5.1	4.9	4.9	4.9	3.4	3.4	3.3	3.8
AD	4.8	4.8	4.8	5.0	5.1	5.0	4.9	5.0	3.1	3.3	3.7	3.9
BD	47.4	60.9	70.6	78.0	48.6	61.6	70.9	78.3	35.9	47.8	57.4	66.0
CD	4.7	4.8	5.1	4.9	5.1	5.0	5.2	5.0	3.2	3.4	3.8	3.9
AE	4.4	4.8	5.0	5.0	4.7	5.0	5.1	5.1	3.0	3.5	3.8	3.8
BE	4.8	4.7	4.8	5.0	5.2	4.9	4.8	5.1	3.3	3.2	3.5	3.9
CE	4.3	4.9	5.0	4.9	4.7	5.0	5.1	5.0	3.0	3.2	3.6	3.7
DE	4.5	4.8	5.0	5.0	4.9	4.9	5.2	5.0	3.1	3.4	3.9	3.7

Table 2. Percentage of rejection of the null hypothesis $H_0 : \alpha_l = 0$ at the 5% level for model $y_{ij} \sim N(5 + 0.15A + 0.15B + 0.15D + 0.12BD, 1)$ in replicated 2^6 experiments.

Effect	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Our method				WH method				Lenth's method			
A	53.3	66.4	76.6	83.8	54.0	66.7	76.8	83.9	47.8	59.7	70.4	78.8
B	53.5	67.0	76.2	83.2	54.3	67.3	76.3	83.2	48.1	60.3	70.3	78.1
C	4.7	5.2	5.1	4.9	4.9	5.3	5.2	4.9	4.1	4.2	4.2	4.0
D	53.1	66.8	75.8	83.1	53.8	67.0	75.9	83.2	47.7	60.0	69.7	77.7
E	5.1	5.2	4.9	5.1	5.3	5.3	5.0	5.1	4.2	4.2	4.2	4.5
F	4.9	5.1	4.9	4.8	5.2	5.2	4.9	4.9	4.3	4.2	4.0	4.3
AB	5.0	5.0	5.0	5.2	5.2	5.1	5.1	5.3	4.1	4.1	4.3	4.5
AC	4.8	5.1	5.0	5.4	5.0	5.2	5.0	5.5	4.0	4.0	4.2	4.6
BC	5.1	5.0	5.0	4.8	5.3	5.0	5.1	4.8	4.2	4.0	4.3	4.2
AD	4.6	5.1	5.1	4.9	4.9	5.1	5.1	5.0	3.8	4.1	4.3	4.3
BD	37.2	48.3	57.2	64.9	37.8	48.6	57.3	65.1	32.4	42.4	51.0	58.7
CD	4.7	4.9	5.1	5.0	5.0	5.0	5.1	5.0	3.9	4.1	4.2	4.2
AE	4.8	5.1	4.9	5.0	5.0	5.2	4.9	5.0	4.0	4.3	4.1	4.1
BE	4.8	5.2	5.0	5.1	5.0	5.3	5.0	5.2	4.1	4.3	4.2	4.4
CE	4.9	4.7	4.9	5.0	5.1	4.8	4.9	5.0	4.2	3.9	4.1	4.4
DE	4.8	5.0	5.2	4.6	5.0	5.1	5.2	4.6	4.2	4.0	4.2	4.0
AF	4.8	5.0	4.9	5.0	4.9	5.1	4.9	5.1	4.0	4.2	4.1	4.3
BF	4.9	5.0	5.0	5.0	5.1	5.0	5.1	5.0	4.1	4.2	4.2	4.2
CF	4.7	4.8	5.1	4.9	4.9	4.9	5.2	4.9	4.1	4.0	4.3	4.2
DF	4.8	4.8	4.8	5.1	5.0	4.9	4.8	5.1	4.2	3.8	4.0	4.3
EF	5.0	5.0	5.0	5.1	5.2	5.1	5.0	5.1	4.3	4.1	4.2	4.3

From the results in Tables 1 and 2, we observe that both our method and the WH method can tightly control the IER at the 5% nominal level when the σ_i^2's are homogeneous. However, Lenth's method is unable to tightly control the IER. In terms of power, our method has almost the same power as the WH method in every case.

Case II: σ_i^2 heterogeneous

In this case, we use the model

$$y_{ij} \sim N\Big(10 + 0.5A + 0.45B + 0.5D + 0.4BD, \exp(1.2A + 1.2B + 1.2D + 0.6AD)\Big)$$

for the 2^5 factorial experiment with five two-level factors, A, B, C, D, and E. For the 2^6 experiment with six two-level factors A, B, C, D, E, and F, we use the model

$$y_{ij} \sim N\Big(10 + 0.4A + 0.3B + 0.4D + 0.3BD, \exp(1.2A + 1.2B + 1.2D + 0.6AD)\Big).$$

We also test the significance of the I factorial effects of interest at the 5% level based on the above methods. For $l = 1, \ldots, I$, the percentage of rejection of the null hypothesis $H_0 : \alpha_l = 0$ at the 5% level by each method is calculated based on $N = 20,000$ repetitions. The results are summarized in Tables 3 and 4. Again, we present only the results for the main effects and two-factor interaction effects because the results for three-factor or higher-order interactions are similar to the results for the effects that are not in the model.

Table 3. Percentage of rejection of the null hypothesis $H_0 : \alpha_l = 0$ at the 5% level for model $y_{ij} \sim N\big(10 + 0.5A + 0.45B + 0.5D + 0.4BD, \exp(1.2A + 1.2B + 1.2D + 0.6AD)\big)$ in replicated 2^5 experiment.

Effect	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Our method				WH method				Lenth's method			
A	28.0	37.8	48.2	56.8	37.0	44.2	53.2	60.5	26.5	33.7	39.6	45.7
B	23.5	31.8	40.3	48.4	31.6	37.9	45.1	52.4	22.7	28.3	34.0	39.7
C	4.4	4.6	4.7	5.1	7.1	6.2	5.9	6.1	2.5	2.6	2.6	2.6
D	27.7	37.7	48.0	56.8	36.7	44.3	53.0	60.5	26.4	32.9	39.3	45.4
E	4.3	4.8	4.8	4.7	7.0	6.5	6.2	5.8	2.8	2.8	2.6	2.7
AB	4.5	4.9	4.8	4.8	7.1	6.7	6.1	5.8	6.9	7.0	6.5	6.5
AC	4.3	4.3	4.8	5.1	7.1	6.0	6.1	6.1	2.6	2.5	2.5	2.7
BC	4.3	4.5	4.8	4.9	7.0	6.2	6.1	6.0	2.5	2.3	2.6	2.4
AD	4.4	4.5	4.6	4.9	7.0	6.3	5.9	5.8	7.2	7.3	6.4	6.7
BD	19.3	26.2	33.2	40.2	26.6	31.6	37.7	43.8	18.6	23.9	28.7	34.0
CD	4.4	4.6	4.5	4.9	6.9	6.4	5.9	6.1	2.5	2.6	2.5	2.6
AE	4.3	4.6	4.6	4.5	6.8	6.5	5.9	5.6	2.6	2.6	2.6	2.6
BE	4.3	4.5	4.9	4.8	6.8	6.4	6.1	5.8	2.5	2.7	2.5	2.8
CE	4.7	4.6	4.9	4.8	7.3	6.4	6.1	5.8	2.7	2.7	2.4	2.6
DE	4.4	4.7	4.8	4.8	6.9	6.4	6.1	5.9	2.6	2.6	2.5	2.7

From the simulated results in Tables 3 and 4, we see that only our method can tightly control the IER in all cases for both models. Again, Lenth's method is unable to accurately control the IER in the location model. The results are quite consistent with those for 2^3 and 2^4 experiments.

We now compare the performance of our method, the WH method, and Lenth's method for controlling the EER in the location model. We again consider two cases: homogeneous σ_i^2's and heterogeneous σ_i^2's.

Case I: σ_i^2 homogeneous

Here, we consider 2^5 and 2^6 factorial experiments. We use the model

$$y_{ij} \sim N(0, 1)$$

for the simulations for both factorial experiments. The simulated EER at the 5% level for each method is calculated based on $N = 20,000$ repetitions. The results are shown

Table 4. Percentage of rejection of the null hypothesis $H_0 : \alpha_l = 0$ at the 5% level for model $y_{ij} \sim N\big(10 + 0.4A + 0.3B + 0.4D + 0.3BD, \exp(1.2A + 1.2B + 1.2D + 0.6AD)\big)$ in replicated 2^6 experiment.

Effect	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Our method				WH method				Lenth's method			
A	37.8	50.6	60.1	69.5	43.9	54.3	62.7	71.4	37.5	47.3	53.9	61.6
B	22.7	31.8	37.9	45.5	27.8	35.1	40.4	47.4	23.5	30.9	35.6	41.5
C	4.4	4.6	4.8	5.2	6.2	5.6	5.6	5.8	3.8	3.7	3.9	4.0
D	37.4	50.6	60.2	69.4	43.6	54.5	62.7	71.1	37.3	46.8	53.3	61.6
E	4.5	4.7	4.6	4.9	6.2	5.8	5.4	5.3	3.8	4.0	3.5	3.6
F	4.5	4.6	4.8	5.0	6.2	5.6	5.6	5.5	3.8	3.8	3.9	3.8
AB	4.5	4.7	4.7	5.0	6.1	5.8	5.6	5.5	6.3	6.5	6.5	6.4
AC	4.6	4.6	4.8	5.2	6.1	5.6	5.6	5.7	4.0	3.8	4.0	4.0
BC	4.6	4.6	5.0	5.0	6.1	5.7	5.7	5.6	3.7	3.8	3.8	4.0
AD	4.5	4.4	4.7	4.7	6.3	5.4	5.4	5.3	6.6	6.3	6.3	5.9
BD	22.7	31.4	37.6	45.5	27.5	34.7	40.1	47.5	23.5	30.8	35.3	41.6
CD	4.5	4.6	5.0	5.2	6.2	5.6	5.8	5.7	4.0	3.9	3.8	4.0
AE	4.5	4.6	4.5	4.8	6.3	5.7	5.2	5.4	3.9	3.9	3.7	3.6
BE	4.5	4.7	4.7	4.5	6.4	5.8	5.4	5.1	4.0	4.0	3.7	3.5
CE	4.5	4.8	4.6	4.8	6.3	5.9	5.4	5.3	4.0	3.9	3.8	3.9
DE	4.5	4.8	4.6	4.8	6.1	5.8	5.4	5.3	3.8	3.9	3.7	3.8
AF	4.4	4.5	4.6	4.8	6.1	5.6	5.4	5.5	3.9	3.8	3.8	3.8
BF	4.2	4.4	4.9	5.0	5.9	5.3	5.7	5.5	3.9	3.8	3.6	3.9
CF	4.5	4.2	5.0	4.9	6.1	5.2	5.7	5.4	3.9	3.6	4.0	4.0
DF	4.3	4.6	5.1	5.0	6.2	5.6	5.8	5.6	3.8	4.0	4.0	3.8
EF	4.6	4.9	4.7	5.0	6.4	5.9	5.5	5.5	3.8	4.0	3.9	3.7

in Table 5. The table shows that all three methods can accurately control the EER at the 5% level.

Table 5. Percentage of rejection of the null hypothesis $H_0 : \alpha_1 = \ldots = \alpha_I = 0$ at the 5% level for model $y_{ij} \sim N(0,1)$ in replicated 2^5 and 2^6 experiments.

I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Our method				WH method				Lenth's method			
31	4.1	4.4	4.9	4.8	5.1	5.0	5.2	5.0	4.9	5.2	5.2	4.9
63	4.3	4.7	4.7	4.8	5.1	5.0	4.9	5.0	4.9	5 5	5.1	

Case II: σ_i^2 heterogeneous

Here, we use the models

$$y_{ij} \sim N\Big(0, \exp(A + C + D + 0.5CD)\Big)$$

and

$$y_{ij} \sim N\Big(0, \exp(A + B + C + D + 0.5AD)\Big)$$

for the 2^5 and 2^6 factorial experiments, respectively. For each method, the EER is calculated based on 20,000 repetitions. The results are summarized in Table 6.

Table 6 shows that only our method can tightly control the EER at the 5% nominal level in all cases. The results given by the WH method are anticonservative when $n = 3$ while those of Lenth's method are slightly conservative.

Table 6. Percentage of rejection of the null hypothesis $H_0 : \alpha_1 = \ldots = \alpha_I = 0$ at the 5% level for models $y_{ij} \sim N\big(0, \exp(A + C + D + 0.5CD)\big)$ and $y_{ij} \sim N\big(0, \exp(A + B + C + D + 0.5AD)\big)$ in replicated 2^5 and 2^6 experiments.

I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	Our method				WH method				Lenth's method			
31	4.6	4.7	4.6	4.7	7.4	6.0	5.1	4.6	4.2	4.3	4.4	4.1
63	4.4	4.5	4.5	4.5	6.9	5.6	4.8	4.2	3.9	3.8	3.9	3.7

1.2. Additional Simulation Results for Dispersion Model

We first compare the performance of our method, the WH method, the VCA method, and Lenth's method for controlling the IER in the dispersion model. In the simulation, we considered 2^5 and 2^6 factorial experiments. For a 2^5 experiment, we generate the data using the model

$$y_{ij} \sim N\Big(0, \exp(0.4A + 0.4B + 0.4C + 0.3AD)\Big).$$

We test the significance of the $I = 2^5 - 1 = 31$ factorial effects of interest at the 5% level based on the above procedures. For $l = 1, \ldots, I$, the percentage of rejection of the null hypothesis $H_0 : \gamma_l = 0$ at the 5% level for each method is calculated based on $N = 20,000$ repetitions. The results are summarized in Table 7.

For a 2^6 factorial experiment, we use the model

$$y_{ij} \sim N\Big(0, \exp(0.3A + 0.3B + 0.3C + 0.26AD)\Big).$$

We test the significance of the $I = 2^6 - 1 = 63$ effects at the 5% level based on all four methods. The simulation is repeated $N = 20,000$ times, and the percentage of each factorial effect declared significant at the 5% level is recorded in Table 8. For both Tables 7 and 8, we present only the results for the main effects and two-factor interaction effects because the results for three-factor or higher-order interactions are similar to the results for the effects that are not in the model.

Tables 7 and 8 show that our method achieves simulated IERs for the factorial effects not in the models that are quite close to the 5% nominal level. The WH method inflates the IER, especially for small n; it becomes better as n increases. Lenth's method is quite conservative whether n is large or small, and the VCA method is slightly anticonservative. The performance is the same for all values of n considered.

We now compare the performance of our method, the WH method, and Lenth's method for controlling EER in the dispersion model. In the simulation, we considered 2^5 and 2^6 factorial experiments. For each experiment, the model under the null hypothesis $H_0 : \gamma_1 = \ldots = \gamma_I = 0$ is

$$y_{ij} \sim N(0, 1).$$

We set the mean of the response to 0, since it does not affect the above three methods. The simulated EER at the 5% level in the dispersion model is calculated based on $N = 20,000$ repetitions. The results are presented in Table 9.

Table 9 shows that the values for the EER based on our method are around 5%. This is evidence that our method can accurately control the EER in the dispersion model. The WH method gives results that are well above the 5% nominal level, so this

Table 7. Percentage of rejection of the null hypothesis $H_0 : \gamma_l = 0$ at the 5% level for model $y_{ij} \sim N\big(0, \exp(0.4A + 0.4B + 0.4C + 0.3AD)\big)$ in replicated 2^5 experiments.

Effect	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	WH method				Our method			
A	59.5	75.6	86.7	92.7	42.2	65.1	80.8	89.7
B	59.5	75.6	86.6	93.0	42.5	65.0	80.8	90.1
C	60.1	75.5	86.2	92.7	42.5	64.8	80.4	89.9
D	12.3	9.5	8.6	7.4	4.8	4.8	5.2	4.9
E	12.6	10.0	8.6	7.6	5.0	5.1	5.1	5.0
AB	12.5	10.1	8.7	7.8	5.1	5.3	5.1	4.9
AC	13.1	9.8	8.6	7.8	5.3	5.0	5.2	5.2
BC	12.4	9.7	8.4	7.5	5.0	5.1	5.1	4.9
AD	41.6	53.9	64.8	74.3	25.8	42.0	56.1	67.7
BD	12.3	9.6	8.2	7.5	5.0	5.2	4.8	5.0
CD	12.4	9.7	8.2	7.7	5.1	5.1	5.0	5.2
AE	12.8	9.7	8.2	7.8	5.1	5.1	5.0	5.1
BE	12.6	9.5	8.4	7.8	4.9	4.8	5.0	5.2
CE	12.8	9.4	8.2	7.7	5.2	4.9	4.8	5.1
DE	12.7	9.8	8.3	7.7	5.2	5.1	4.9	5.2
	Lenth's method				VCA method			
A	32.4	50.4	66.5	78.6	41.9	64.4	79.9	89.3
B	32.4	50.8	66.1	78.8	41.9	64.0	79.7	89.7
C	32.5	50.2	65.9	78.4	42.2	63.9	79.4	89.4
D	2.9	2.9	3.4	3.5	5.3	5.5	5.8	5.5
E	2.9	3.2	3.5	3.5	5.7	5.6	5.7	5.5
AB	3.2	3.1	3.4	3.4	5.6	5.8	5.6	5.4
AC	3.2	3.0	3.5	3.5	5.9	5.7	5.7	5.7
BC	3.0	3.2	3.4	3.5	5.6	5.6	5.7	5.4
AD	18.8	30.7	42.7	54.1	26.1	41.4	55.6	67.4
BD	3.2	3.1	3.1	3.4	5.5	5.9	5.3	5.4
CD	3.2	3.0	3.2	3.6	5.6	5.5	5.5	5.7
AE	3.2	3.1	3.4	3.7	5.6	5.5	5.6	5.7
BE	3.2	3.1	3.2	3.5	5.6	5.5	5.3	5.8
CE	3.1	3.2	3.3	3.5	5.8	5.4	5.5	5.7
DE	3.3	3.0	3.4	3.5	5.6	5.6	5.4	5.7

method cannot control the EER. The EER based on Lenth's method is quite close to the nominal level, so Lenth's method can also tightly control the EER.

2. R function *rrtff()*

We have written an R function *rrtff()* to implement the proposed methods for the 2^k full factorial experiment. The input of *rrtff()* has four parts:

(1) *ymat*: a $2^k \times n$ numeric matrix with each row consisting of n replications of the response;
(2) *xmat*: a $2^k \times k$ numeric matrix, which consists of all 2^k combinations of the k two-level factors (we use "-1" and "+1" to denote the two levels);
(3) *level*: the significance level, with the default value 0.05;
(4) *type*: *type=location* indicates the location model; *type=dispersion* indicates the dispersion model.

If *type=location*, then the R function returns the $\hat{\alpha}_l$'s (*alp* in the R output), t-type statistics t_l's (*tstat*), C_{IER} (*CIER*), and C_{EER} (*CEER*). If *type=dispersion*, then it returns the $\hat{\gamma}_l$'s (*gam* in the R output), z-type statistics z_l's (*zstat*), C_{IER} (*CIER*), and

Table 8. Percentage of rejection of the null hypothesis $H_0 : \gamma_l = 0$ at the 5% level for model $y_{ij} \sim N(0, \exp(0.3A + 0.3B + 0.3C + 0.25AD))$ in replicated 2^6 experiments.

Effect	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	WH method				Our method			
A	63.5	79.6	89.5	95.2	46.2	70.0	84.8	93.1
B	63.9	79.6	89.5	95.3	46.6	69.6	84.7	93.1
C	64.3	79.5	89.5	95.0	47.3	69.8	84.8	92.8
D	12.8	9.6	8.3	7.7	5.2	4.9	4.9	4.9
E	12.7	10.1	8.5	7.8	5.0	5.2	5.1	4.9
F	12.6	10.1	8.4	7.4	5.1	5.0	5.1	4.8
AB	12.7	9.6	8.0	7.8	4.8	5.0	4.7	5.1
AC	12.5	9.8	8.5	7.7	5.1	5.0	5.1	5.1
BC	12.9	9.7	8.8	7.8	5.2	4.7	5.1	5.0
AD	53.9	69.0	80.4	88.7	36.6	57.9	73.3	84.5
BD	13.2	10.1	8.4	7.6	5.2	5.1	5.1	4.9
CD	12.7	9.8	8.2	7.7	5.1	5.2	4.7	4.9
AE	12.8	10.0	8.5	7.7	5.2	5.2	5.1	5.2
BE	12.7	9.8	8.2	7.4	5.1	5.1	4.8	4.9
CE	12.6	9.7	8.2	7.5	5.1	5.1	4.9	4.9
DE	12.7	10.1	8.5	7.7	4.9	5.1	5.1	5.1
AF	12.9	9.6	8.5	7.6	5.1	4.8	4.9	5.0
BF	12.6	9.9	8.7	7.8	5.1	5.1	5.3	5.1
CF	12.2	9.6	8.3	7.6	4.8	4.9	4.8	4.9
DF	12.4	9.7	8.2	7.4	4.9	4.9	5.0	4.7
EF	12.4	9.8	8.4	8.0	4.7	5.0	5.0	5.1
	Lenth's method				VCA method			
A	41.0	63.1	79.1	89.2	46.4	69.6	84.3	92.9
B	40.5	62.7	79.3	89.4	46.3	69.2	84.2	92.9
C	41.2	62.9	79.2	89.4	47.1	69.0	84.2	92.6
D	4.1	4.1	4.1	4.3	5.5	5.2	5.3	5.3
E	3.9	4.2	4.3	4.4	5.3	5.4	5.4	5.4
F	3.8	4.0	4.0	4.0	5.4	5.3	5.1	5.1
AB	3.8	4.1	4.1	4.4	5.2	5.3	4.8	5.5
AC	3.8	4.0	4.3	4.5	5.5	5.3	5.3	5.5
BC	4.1	3.7	4.2	4.3	5.4	5.0	5.5	5.4
AD	31.5	51.0	67.0	79.4	36.7	57.3	72.9	84.4
BD	4.1	4.1	4.1	4.3	5.5	5.6	5.4	5.4
CD	3.9	4.1	4.0	4.2	5.5	5.5	4.9	5.1
AE	3.9	4.1	4.3	4.5	5.5	5.4	5.4	5.4
BE	3.8	4.0	4.0	4.4	5.3	5.4	5.3	5.2
CE	3.8	4.0	4.1	4.2	5.3	5.3	5.1	5.3
DE	3.7	4.1	4.2	4.4	5.4	5.3	5.4	5.3
AF	3.9	3.9	4.3	4.5	5.5	5.1	5.2	5.3
BF	3.9	4.1	4.2	4.4	5.5	5.4	5.5	5.4
CF	3.8	4.0	4.0	4.3	5.3	5.2	5.2	5.2
DF	3.8	3.9	4.0	4.2	5.5	5.2	5.2	5.0
EF	3.8	3.9	4.1	4.6	5.1	5.1	5.3	5.5

Table 9. Percentage of rejection of the null hypothesis $H_0 : \gamma_1 = \ldots = \gamma_I = 0$ at the 5% level for model $y_{ij} \sim N(0, 1)$ in replicated 2^5 and 2^6 factorial experiments.

I	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$	$n=3$	$n=4$	$n=5$	$n=6$
	WH method				Our method				Lenth's method			
31	33.0	20.8	15.7	13.3	5.5	5.5	5.4	5.3	4.0	4.4	4.6	4.5
63	40.9	24.6	18.3	14.4	5.5	5.2	5.3	5.3	4.1	4.4	4.7	4.6

C_{EER} (*CEER*).

The R function *rrtff()* is as follows.

```
library("MASS")
rrtff=function(ymat,xmat,level=0.05,type="location")
{
xmat=as.matrix(data.frame(xmat))
ymat=as.matrix(data.frame(ymat))
m=nrow(ymat)
n=ncol(ymat)

ybar=as.numeric(apply(ymat,1,mean))
s2=as.numeric(apply(ymat,1,var))+1e-100
cnm=colnames(xmat)
lns2=log(s2)

if(type=="location")
{
M=100000
expr1=paste("ybar~",paste(cnm,collapse="*"),sep="")
data=data.frame(cbind(ybar,xmat))

out.loc=lm(eval(expr1),data=data,x=T)
alp=out.loc$coefficients[-1]
alp.se=sqrt(sum(s2)/(m^2*n))
tstat=(alp)/alp.se

rho=s2/sum(s2)
mx=out.loc$x
mx=as.matrix(mx)[,-1]

Amat=t(mx)%*%diag(rho)%*%mx
Umat=mvrnorm(M,rep(0,nrow(Amat)),Amat)
Vmat=matrix(rchisq(M*m,n-1),ncol=m)
tb=abs(Umat[,1])/sqrt( (Vmat%*%rho)/(n-1) )
tb=as.numeric(tb)
maxtb=apply(abs(Umat),1,max)/sqrt( (Vmat%*%rho)/(n-1) )
maxtb=as.numeric(maxtb)

out=list(alp=alp,tstat=tstat,CIER=quantile(tb,1-level),
            CEER=quantile(maxtb,1-level))
}
```

```
if(type=="dispersion")
{
expr2=paste("lns2~",paste(cnm,collapse="*"),sep="")
data=data.frame(cbind(lns2,xmat))
out=lm(eval(expr2),data=data)
gam=out$coefficients[-1]
gam.se=sqrt(2/(m*(n-1)))
zstat=gam/gam.se

f1=function(x)
{
log(x)*dchisq(x,df=n-1)
}

f2=function(x)
{
( log(x) )^2*dchisq(x,df=n-1)
}

mom1=integrate(f1,0,Inf)
mom2=integrate(f2,0,Inf)
an.var=mom2$value-mom1$value^2

an=sqrt(an.var/(2/(n-1)))
Ieff=length(zstat)

out=list(gam=gam,zstat=zstat, CIER=an*qnorm(1-level/2),
            CEER=an*qnorm(0.5+0.5*(1-level)^(1/Ieff) ) )
}
out
}
```

The following is the R code that inputs the golf data used in Section 5 of the main paper.

```
data=c(-1,-1,-1,-1,10,18,14,12.5,19.0,16.0,18.5,
+1,-1,-1,-1,0,16.5,4.5,17.5,20.5,17.5,33,
-1,+1,-1,-1,4,6,1,14.5,12,14,5,
+1,+1,-1,-1,0,10,34,11,25.5,21.5,0,
-1,-1,+1,-1,0,0,18.5,19.5,16,15,11,
+1,-1,+1,-1,5,20.5,18,20,29.5,19,10,
-1,+1,+1,-1,6.5,18.5,7.5,6,0,10,0,
+1,+1,+1,-1,16.5,4.5,0,23.5,8,8,8,
-1,-1,-1,+1,4.5,18,14.5,10,0,17.5,6,
+1,-1,-1,+1,19.5,18,16,5.5,10,7,36,
-1,+1,-1,+1,15,16,8.5,0,0.5,9,3,
+1,+1,-1,+1,41.5,39,6.5,3.5,7,8.5,36,
-1,-1,+1,+1,8,4.5,6.5,10,13,41,14,
+1,-1,+1,+1,21.5,10.5,6.5,0,15.5,24,16,
-1,+1,+1,+1,0,0,0,4.5,1,4,6.5,
+1,+1,+1,+1,18.5,5,7,10,32.5,18.5,8)
```

```
data=matrix(data,nrow=16,byrow=T)
xt=data[,1:4]
yt=data[,-(1:4)]
```

We give below the R code and output for the analysis of the location model of the golf data. If we control the IER at 5%, then the main effects A and B are seen to be significant by comparing the t-type statistics and C_{IER} from the output. If we control the EER at 5%, then only effect A is significant.

```
> rrtff(yt,xt,0.05,"location")
$alp
        X1         X2         X3         X4      X1:X2      X1:X3
 2.8660714 -1.8571429 -1.1339286 -0.1071429  1.4017857 -0.3214286
     X2:X3      X1:X4      X2:X4      X3:X4   X1:X2:X3   X1:X2:X4
-1.0089286  0.9196429  0.7142857 -0.1160714 -0.2500000  1.0089286
  X1:X3:X4   X2:X3:X4 X1:X2:X3:X4
-0.5892857 -0.5446429  0.9285714

$tstat
        X1         X2         X3         X4      X1:X2      X1:X3
 3.2581697 -2.1112128 -1.2890578 -0.1218007  1.5935596 -0.3654022
     X2:X3      X1:X4      X2:X4      X3:X4   X1:X2:X3   X1:X2:X4
-1.1469569  1.0454563  0.8120049 -0.1319508 -0.2842017  1.1469569
  X1:X3:X4   X2:X3:X4 X1:X2:X3:X4
-0.6699041 -0.6191537  1.0556064

$CIER
     95%
1.994946

$CEER
     95%
3.006521
```

We give below the R code and output for the analysis of the dispersion model of the golf data. If we control the IER at 5%, then the effects A and BC are seen to be significant by comparing the z-type statistics and C_{IER} from the output. If we control the EER at 5%, then only effect A is significant.

```
> rrtff(yt,xt,0.05,"dispersion")
$gam
        X1          X2          X3          X4       X1:X2       X1:X3
 0.56085423 -0.07094417 -0.10816421  0.12444623  0.27955487 -0.29552695
     X2:X3       X1:X4       X2:X4       X3:X4    X1:X2:X3    X1:X2:X4
-0.34700280  0.01005715 -0.16036735 -0.16803847  0.20423514  0.28474751
  X1:X3:X4    X2:X3:X4 X1:X2:X3:X4
 0.17318467 -0.11905612  0.07103150

$zstat
        X1          X2          X3          X4       X1:X2       X1:X3
```

```
 3.88571207 -0.49151559 -0.74938362  0.86218876  1.93681296 -2.04747076 
      X2:X3       X1:X4       X2:X4       X3:X4    X1:X2:X3    X1:X2:X4 
-2.40410590  0.06967796 -1.11105760 -1.16420467  1.41498255  1.97278860 
   X1:X3:X4    X2:X3:X4 X1:X2:X3:X4 
 1.19985862 -0.82484501  0.49212069 

$CIER
[1] 2.133394

$CEER
[1] 3.186868
```