CO 439/739 Combinatorial Algebraic Geometry
Fall 2025

Commutative algebra and **algebraic geometry** have a reputation for being difficult and unintuitive, even though they are both at heart just the study of polynomials. We will study these areas through important families of examples, where the mysteries can be rendered concrete and combinatorial. By the end of the course, you should have a solid grounding for further study of the more arcane parts of commutative algebra and algebraic geometry, as well as the ability to tackle open problems of a combinatorial flavour.

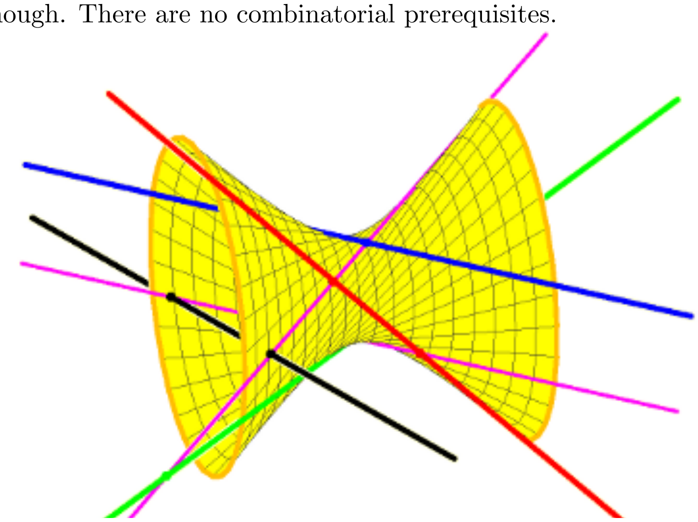
Our main tool will be the theory of **Gröbner bases**, which is often presented as just a computational device for working with ideals of commutative rings. Indeed, Gröbner bases are great computational devices and we'll learn how to compute with them. But we can also use Gröbner bases both to prove general theorems and to connect to rich combinatorial structures!

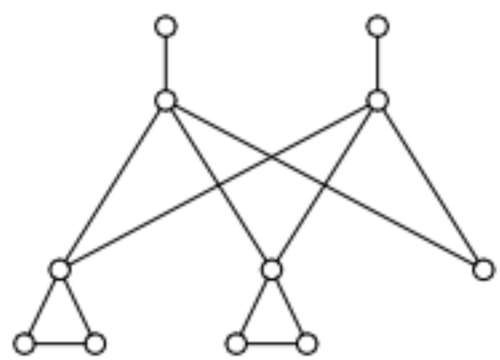
Topics touched on will include

- Determinantal varieties, Grassmannians, toric varieties, flag varieties;
- Simplicial complexes, Stanley–Reisner ideals;
- Schur polynomials, Schubert polynomials, Grothendieck polynomials;
- Hilbert series, Betti numbers, regularity, etc!

If you already know what these words mean, you'll learn new ways to think about them; if they are new to you, you'll learn that they can be your friends.

Pre-/Co-requisite for undergraduates: PMATH 334 (Introduction to Rings and Fields with Applications) or PMATH 347 (Groups and Rings). Most of the algebra we need, we will build during the course. You should have seen commutative rings before and be comfortable quotienting them by ideals, but that's probably enough. There are no combinatorial prerequisites.





Images above shamelessly stolen from Frank Sottile (left) and Hibi-Kimura-Matsuda-Van Tuyl (right).