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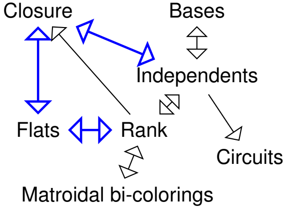
Constructions of new matroids and designs over Gf(q)

q-matroid: generalization replacing finite sets by *Gf(q)-vector spaces*.
q-design $t-(n,k,\lambda;q)$; $\lambda = 1$**:** q-Steiner system S(t, k, n; q).
Existence for $t > 1$**?** S(2, 3, 13; 2); **smallest open case:** S(2, 3, 7; q): q-analogue of the *Fano plane.*
q-PMD: q-matroid s.t. any two flats of the same rank have the same dimension.

q-cryptomorphisms



q-Steiner systems are q-PMDs

M: q-PMD induced by S(t, k, n; q):
Independents dim. $t+1$: $t-(n,t+1,\lambda_I)$,
$\lambda_I = (q^{n-t}-q^{k-t})/(q-1)$;
Circuits: dim. $t+1$: $t-(n,t+1,\lambda_{C_{t+1}})$,
$\lambda_{C_{t+1}} = \begin{bmatrix} k-t \\ 1 \end{bmatrix}_q$;
dim. $t+2$: $t-(n,t+2,\lambda_{C_{t+2}})$,
$\lambda_{C_{t+1}} = q^{k-t}\begin{bmatrix} n-k \\ 1 \end{bmatrix}_q \left(\begin{bmatrix} n-t-1 \\ 1 \end{bmatrix}_q - \frac{1}{q}\begin{bmatrix} k-t \\ 1 \end{bmatrix}_q \begin{bmatrix} t+1 \\ 1 \end{bmatrix}_q\right)\frac{1}{q+1}$

Automorphism group of S(t,k,n;q) isomorphic to that of $t-(n,t+1,\lambda_I)$ and $t-(n,t+2,\lambda_{C_{t+2}}) \rightarrow$ the known subspace designs by Braun et al. cannot be derived from the q-Fano plane via our construction.