

# WORKSHOP ON NUMBER THEORY AND RANDOM MATRIX THEORY

WATERLOO, ONTARIO

JUNE 1 – 3, 2005

ORGANIZERS: YU-RU LIU, DAVID MCKINNON  
AND MICHAEL RUBINSTEIN



## ACKNOWLEDGEMENTS

The organizers of the Workshop on Number Theory and Random Matrix Theory would like to take this opportunity to thank the *Fields Institute*, the *Faculty of Mathematics*, *PIMS*, the *Perimeter Institute* and the *Department of Pure Mathematics* at the University of Waterloo for their generous support. Finally, research funds from individual participants continue to play an essential role in helping to fund the conference, and helping to defray the costs for graduate students and postdoctoral fellows with no other means of support. We are very grateful to those who were able to contribute in this manner.

It is hoped that during your stay you will have the opportunity to venture out beyond the confines of the campus to explore a little of Waterloo. You will find a list of some restaurants enclosed in this registration package.

Finally, the organizers would like to thank Ms. Kimberley Gingerich, Ms. Lis D'Alessio and Ms. Shonn Martin for all of their help in the organization of the conference. They did a marvelous job and we certainly could not have done it without them.



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## Program Notes.

Welcome to the University of Waterloo, and in particular to the Workshop on Number Theory and Random Matrix Theory. Please take the time to look over this registration booklet as it contains information which may be of use to you during your stay.

- A CAMPUS MAP has been provided in the registration package. Here are the references:
  - *Ron Eydt Village*, the University Residences are located in square B 1.
  - The Pure Mathematics Department is located on the fifth floor of the *Math & Computer Building*, also known as *MC* (see E 2 on the map).
- - Yu-Ru Liu's office is MC 5074. Phone number: 888-4567 x-5698.
  - David McKinnon's office is MC 5167. Phone number: 888-4567 x-5560.
  - Michael Rubinstein's office is MC 5044. Phone number: 888-4567 x-6172
  - The Department of Pure Mathematics is in MC 5066. Phone number: 888-4567 x-3484. Please stop by and talk with Kim Gingerich if you have any questions.
- The University Bookstore and BookEnd Cafe are located in South Campus Hall (SCH) (see E 4 on the map). Summer hours are: 8:00 - 5:00 pm, Monday to Friday and Saturday, 12:00-4:00 pm. The Bookstore is closed on Sundays and statutory holidays.
- The University Shops Plaza (see H 4 on the map) has a large number of restaurants and stores. It is a very convenient place to have lunch.
- **REGISTRATION** will take place on Wednesday, June 1st from 8:00 - 9:00 am in the Math & Computer Building, Room MC 5136 (see E 2 on the map). If you were not able to pick up your package please see Kim Gingerich in MC 5066 during the breaks.
- **TALKS BEGIN on Wednesday, June 1st at 9:00 am.** Talks take place in the Math & Computer Building, Room 5136.

- THURSDAY AFTERNOON has been kept free. In this registration package you will find a “Visitor Brochure” where you can find some suggestions on things to do in Waterloo and the surrounding area.

### Final Schedule

All talks take place in the Math & Computer Building,  
Room 5136

<b>Wednesday, June 1st</b>	
9:00 - 9:50	Chris Hughes
9:50 - 10:40	Jeffrey Lagarias
10:40 - 11:00	<b>Coffee Break</b>
11:00 - 12:00	Eugene Bogomolny
12:00 - 2:00	<b>Lunch</b>
2:00 - 2:50	Pär Kurlberg
2:50 - 3:40	Alex Gamburd
3:40 - 4:00	<b>Coffee Break</b>
4:00 - 4:50	Steven J. Miller

<b>Thursday, June 2nd</b>	
9:00 - 9:40	Nina Snaithe
9:40 - 10:20	David Farmer
10:20 - 10:40	<b>Coffee Break</b>
10:40 - 11:20	Oriol Bohigas
11:20 - 12:00	Francesco Mezzadri
12:00	<b>Free Afternoon</b>

<b>Friday, June 3rd</b>	
9:00 - 9:50	Ram Murty
9:50 - 10:40	Matthew Young
10:40 - 11:00	<b>Coffee Break</b>
11:00 - 12:00	Yiannis Petridis
12:00 - 2:00	<b>Lunch</b>
2:00 - 2:40	Christian Roettger
2:40 - 3:20	Hershy Kisilevsky
3:20 - 3:40	<b>Coffee Break</b>
3:40 - 4:20	Nathan Ng
4:20 - 5:00	Gang Yu

## List of speakers and their abstracts.

**Bogomolny, E.**

*Université Paris-Sud*

SPECTRAL STATISTICS OF A PSEUDO-INTEGRABLE MAP

WEDNESDAY, JUNE 1. 11:00 - 12:00 MC 5136

Spectral properties of special random unitary matrices appearing under quantization of a pseudo-integrable map are investigated. Under certain conditions it is demonstrated that they are described by the semi-Poisson statistics with arbitrary integer or half-integer level repulsion at small distances.

**Bohigas, O.**

*Université Paris-Sud*

SPACING DISTRIBUTION OF ZEROS OF RIEMANN'S ZETA

THURSDAY, JUNE 2. 10:40 - 11:20 MC 5136

Asymptotically the spacing distribution of Riemann's zeros is expected to coincide with the random matrix prediction (Montgomery-Odlyzko). We discuss, heuristically, how this asymptotic result is approached and compare it with numerical data of Odlyzko.

**Farmer, D.**

*American Institute of Mathematics*

EXAMPLE MAASS L-FUNCTIONS

THURSDAY, JUNE 2. 9:40 - 10:20 MC 5136

I will discuss some calculations of Maass forms and their associated  $L$ -functions.

**Gamburd, A.**

*Stanford University*

AVERAGES OF CHARACTERISTIC POLYNOMIALS FROM CLASSICAL GROUPS

WEDNESDAY, JUNE 1. 2:50 - 3:40 MC 5136

We present an elementary and self-contained derivation of formulae for products and ratios of characteristic polynomials from classical groups using classical results due to Weyl and Littlewood. Joint work with Dan Bump.

**Hughes, C.**

*University of Michigan*

THE MAXIMUM SIZE OF THE ZETA FUNCTION

WEDNESDAY, JUNE 1. 9:00 - 9:50 MC 5136

A new conjecture for the true rate of growth of the Riemann zeta function is presented. This is joint work with David Farmer and Steve Gonek.

**Kisilevsky, H.**

*Concordia University*

CENTRAL VALUES OF CUBIC ELLIPTIC L-FUNCTIONS

FRIDAY, JUNE 3. 2:40 - 3:20 MC 5136

This is a report on our work on the central values of twisted  $L$ -functions of elliptic curves over the rationals. We consider twists of  $L(E, s)$  by Artin characters of cubic extensions of  $\mathbf{Q}$ , and present our results, and experimental data.

**Kurlberg, P.**

*KTH*

POISSON STATISTICS VIA THE CHINESE REMAINDER THEOREM

WEDNESDAY, JUNE 1. 2:00 - 2:50 MC 5136

We consider the distribution of spacings between consecutive elements in subsets of  $\mathbb{Z}/q\mathbb{Z}$  where  $q$  is highly composite and the subsets are defined via the Chinese remainder theorem. We give a sufficient criterion for the spacing distribution to be Poissonian as the number of prime factors of  $q$  tends to infinity, and as an application we show that the value set of a generic (Morse) polynomial modulo  $q$  has Poisson spacings. (Joint work with A. Granville.)

**Lagarias, J.***University of Michigan*ZERO SPACING DISTRIBUTIONS FOR DIFFERENCED  $L$ -FUNCTIONS

WEDNESDAY, JUNE 1. 9:50 - 10:40 MC 5136

The GUE conjecture on the local zero spacings of various  $L$ -functions is shown to not survive deformation by a differencing operation. On the Riemann hypothesis these deformations keep all zeros on the critical line, but their spacings become very regular. There likely exist random matrix analogues of these phenomena. The talk will also describe relations to the theory of Hilbert spaces of entire functions.

**Mezzadri, F.***University of Bristol*

RANDOM MATRIX THEORY AND ENTANGLEMENT IN QUANTUM SPIN CHAINS

THURSDAY, JUNE 2. 11:20 - 12:00 MC 5136

We compute the entropy of entanglement in the ground states of a general class of quantum spin-chain Hamiltonians — those that are related to quadratic forms of Fermi operators — between the first  $N$  spins and the rest of the system in the limit of infinite total chain length. We show that the entropy can be expressed in terms of averages over the classical compact groups and establish an explicit correspondence between the symmetries of a given Hamiltonian and those characterizing the Haar measure of the associated group. These averages are either Toeplitz determinants or determinants of combinations of Toeplitz and Hankel matrices. Recent generalizations of the Fisher-Hartwig conjecture are used to compute the leading order asymptotics of the entropy as  $N \rightarrow \infty$ . This is shown to grow logarithmically with  $N$ . The constant of proportionality is determined explicitly, as is the next (constant) term in the asymptotic expansion. The logarithmic growth of the entropy was previously predicted on the basis of numerical computations and conformal-field-theoretic calculations. In these calculations the constant of proportionality was determined in terms of the central charge of the Virasoro algebra. Our results therefore lead to an explicit formula for this charge. We also show that the entropy is related to solutions of ordinary differential equations of Painlevé type. In some cases these solutions can be evaluated to all orders using recurrence relations.

**Miller, S.***Brown University*

IDENTIFYING SYMMETRY GROUPS OF ZEROS OF FAMILIES OF L-FUNCTIONS

WEDNESDAY, JUNE 1. 4:00 - 4:50 MC 5136

Many studies have shown that the zeros near the central point of families of  $L$ -functions are well modelled by eigenvalues near one of a classical compact group. While the monodromy group in the function field case suggests what that group should be, determining the group in the number field case is often a matter of calculation (and sometimes the results seem surprising at first). We investigate the symmetry group of Rankin-Selberg convolutions of known families. The symmetry group of the product can be determined from the symmetries of the constituents. Similar to the universality observed in the high zeros, the symmetry group seems to be controlled by the second moment of the Satake parameters. As an application we study the convolution of one-parameter families of elliptic curves with rank over  $\mathbf{Q}(T)$ .

**Murty, R.***Queen's University*

DIRICHLET SERIES AND HYPERELLIPTIC CURVES, PART 1

FRIDAY, JUNE 3. 9:00 - 9:50 MC 5136

We attach Dirichlet series to hyperelliptic and superelliptic curves and obtain diophantine information from them. This is joint work with Jung-Jo Lee.

**Ng, N.***University of Michigan*

LARGE GAPS BETWEEN THE ZEROS OF THE ZETA FUNCTION

FRIDAY, JUNE 3. 3:40 - 4:20 MC 5136

I will discuss the moment method for finding large and (small) gaps between the zeros of the Riemann function. In particular I will show how GRH can be used to find gaps of size 2.91 times larger than the average spacing.

**Petridis, Y.** *City University of New York and Max-Planck-Institute fuer Mathematik*

MODULAR SYMBOLS AND DISCRETE LOGARITHMS

FRIDAY, JUNE 3. 11:00 - 12:00 MC 5136

We investigate the distribution of the values of additive homomorphisms from a discrete group (like  $\Gamma_0(N)$  or a free group) to  $(C, +)$ . Two interesting cases are the modular symbols and the discrete logarithms. We work with appropriate zeta (generating) functions: Selberg zeta and Ihara zeta functions. This is joint work with Morten S. Risager.

**Roettger, C.**

*Iowa State University*

COUNTING MATRICES IN THE GENERAL LINEAR GROUP OVER ALGEBRAIC INTEGERS

FRIDAY, JUNE 3. 2:00 - 2:40 MC 5136

For a  $2 \times 2$ -matrix  $A$  with entries in the ring  $R$  of algebraic integers of some number field, define the height as the maximum of all conjugates of all entries in absolute value. Let  $N(T)$  be the number of matrices in the special linear group  $SL_2(R)$  whose height is bounded by  $T$ . We use uniform distribution and Kloosterman sums to determine the asymptotic behavior of  $N(T)$  and investigate the same question for the general linear group  $GL_2(R)$ .

**Snaith, N.**

*University of Bristol*

DERIVATIVES OF RANDOM MATRIX CHARACTERISTIC POLYNOMIALS AND APPLICATIONS TO ELLIPTIC CURVES

THURSDAY, JUNE 2. 9:00 - 9:40 MC 5136

Motivated by applications in number theory to the rank of elliptic curves, we calculate the value distribution of derivatives of characteristic polynomials of matrices from  $SO(N)$  at the point 1, the symmetry point on the unit circle of the eigenvalues of these matrices. We consider subsets of matrices from  $SO(N)$  which are constrained to have  $m$  eigenvalues equal to 1 and look at the first non-zero derivative of the characteristic polynomial at that point.

**Young, M.***American Institute of Mathematics*

MOMENTS OF THE CRITICAL VALUES OF FAMILIES OF ELLIPTIC CURVES

FRIDAY, JUNE 3. 9:50 - 10:40 MC 5136

We make a conjecture on the moments of the central values of the family of all elliptic curves. As an interesting application we make a conjecture on the proportion of elliptic curves of rank 2 with proscribed behavior modulo  $p$ . Further, we make a conjecture on the moments of the first derivative of the central values of a large family of positive rank curves. The order of magnitude is the same as the that of the moments of the central values of an orthogonal family.

**Yu, G.***University of South Carolina*

AVERAGE FROBENIUS DISTRIBUTION OF ELLIPTIC CURVES

Joint with Kevin James

FRIDAY, JUNE 3. 4:20 - 5:00 MC 5136

For an elliptic curve  $E/\mathbb{Q}$  and a prime number  $p$ , let

$$a_E(p) := p + 1 - |E(\mathbb{F}_p)|$$

where  $E(\mathbb{F}_p)$  is the set of  $\mathbb{F}_p$ -rational points on  $E$  (along with an *identity* at  $\infty$ , which is an abelian group when  $E$  has good reduction modulo  $p$ .) For real numbers  $-1 \leq \alpha \leq \beta \leq 1$  and  $X > 1$ , let

$$\pi_E^{\alpha, \beta}(X) := \#\left\{p \leq X : \frac{a_E(p)}{2\sqrt{p}} \in (\alpha, \beta]\right\}.$$

The Sato-Tate Conjecture claims that, when  $E$  does not admit complex multiplication, one should have

$$\pi_E^{\alpha, \beta}(X) \sim \frac{2}{\pi} \int_{\alpha}^{\beta} \sqrt{1-t^2} dt \quad \text{as } X \rightarrow \infty.$$

We show that, subject to varying the curves in a certain range, the conjecture is true on average.

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## IMPORTANT LOCAL INFORMATION.

- **Internet Connection**

- While you are here, if you're like most of us, you will want to check your email. Computers are available in the Davis Centre Library (DC).

- **Emergency Information**

- Police, Fire or Ambulance, **911**.
- Grand River Hospital, K-W Health Centre, 835 King Street W, Kitchener: 749-4242 (Emergency Department).
- Poison Information: 1-800-267-1373.
- Pharmacy: On campus - Student Health Pharmacy, Student Life Centre (SLC) lower level: 746-4500 (open Monday-Friday 9-5).
- Pharmacy: Shoppers Drug Mart, Parkdale Plaza, 468 Albert Street, Waterloo: 884-3860 (open Monday-Friday 9-9, Saturday 9-6, Sunday 10-6).
- Pharmacy: Westmount Place Pharmacy (Pharmasave), 50 Westmount N, Waterloo: 886-7670 (open Monday-Friday 9-9, Saturday 9-6, Sunday 11-6).

- **Transportation**

- City Cabs: 747-7777
- United Taxi: 888-9999
- Waterloo Taxi: 886-1200
- Airways Transit: 886-2121
- Grand River Transit, Bus Route Information: 585-7555

- **Banks and ATM Machines**

- ATM Machine: On campus - Davis Centre (DC) main level
- ATM Machine: On campus - Student Life Centre (SLC) main and lower levels
- CIBC, Student Life Centre (SLC) lower level: 884-4760
- CIBC, Conestoga Mall, 550 King N, Waterloo: 884-9230

- BMO Bank of Montreal, 730 Glen Forrest Blvd, Waterloo: 885-8410
- BMO Bank of Montreal, 3 King S, Waterloo: 885-9250
- TD Canada Trust, Westmount Mall, 50 Westmount N, Waterloo: 885-8550
- TD Canada Trust, Conestoga Mall, 550 King N, Waterloo: 885-8485
- Scotiabank, 569 King Street N, Waterloo: 884-2430
- Scotiabank, Waterloo Square, 75 King Street S, Waterloo: 886-2500
- RBC Royal Bank, 74 King Street S, Waterloo: 747-8335
- RBC Royal Bank, King & University, Waterloo: 747-8320

• **Post Office**

- Pak Mail, 160 Columbia W, Waterloo: 746-0202
- Westmount Place Pharmacy (Pharmasave), 50 Westmount N, Waterloo: 886-7670
- Drug Mart, 468 Albert Street, Waterloo: 884-3860

• **Libraries**

- On campus - Davis Centre Library - Engineering, Mathematics, and Physical and Life Sciences
- On campus - Dana Porter Library - Arts, Humanities, and Social Sciences
- Off campus - City of Waterloo Public Library, Main Library, 35 Albert Street, Waterloo: 886-1310
- Off campus - City of Waterloo Public Library, McCormick Branch, 500 Parkside Drive, Waterloo: 885-1920