PMATH 955 -- Topics in Geometry: Generalized Complex Geometry
Course outline − Winter 2012

Instructor: Ruxandra Moraru
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Office hours (MC 5170): TTh 11:00—12:00, or by appointment.

Lectures: TTh 9:00 - 10:20 (MC 5046).

Course webpage: can be found at http://www.math.uwaterloo.ca/~moraru/955_2012.html.

Overview: Generalized complex geometry, which interpolates between symplectic and complex geometry, was introduced by Hitchin in 2002. Its framework has since led to important breakthroughs in bi-Hermitian geometry as well as mirror symmetry. This course will be an introduction to this exciting new research area and will cover material that any graduate student interested in geometry will find useful (such as bundles, connections, sheaf cohomology, symplectic structures, complex structures and their deformations, symplectic reductions, and moduli spaces).

The course should be accessible to students who have taken PMATH 465 (Differential Geometry) or an equivalent course.

Outline of topics:

1. *Background from symplectic, Poisson, and complex geometry:*

– Differential forms, de Rham cohomology, and Dolbeault cohomology.
– Riemannian metrics, volume forms, and the Hodge star operator.
– Vector bundles and connections: definitions and basic constructions.
– Symplectic vector spaces, Lagrangian subspaces, and symplectic bundles.
– Symplectic manifolds, Darboux's theorem, and submanifolds of symplectic manifolds.
– Poisson algebras and Poisson manifolds.
– Hamiltonian group actions, moment maps, and symplectic reductions *(time permitting)*.
– Almost complex structures, integrability, and Kähler manifolds.
– Sheaf cohomology and deformations of complex structures.

2. *Generalized complex geometry:*

– Courant brackets and B-fields; generalized metrics and the generalized Hodge operator.
– Gerbes, exact Courant algebroids, and twisted structures.
– Generalized complex manifolds: definitions and basic examples.
– Integrability and spinors; generalized Calabi-Yau manifolds.
– The generalized Darboux theorem; generalized complex submanifolds.
– Deformations of generalized complex structures.
– Generalized Kähler manifolds and their relation to bi-Hermitian geometry.
– Lie algebroid connections and generalized holomorphic vector bundles.
– Reductions of generalized complex and generalized Kähler structures *(time permitting)*.
– Construction of new examples of generalized complex and generalized Kähler spaces via reductions methods *(time permitting)*.

Some references:

1. M. Gualtieri, *Generalized complex geometry*, arXiv:math/0401221v1.
2. N. Hitchin, *Lectures on generalized geometry*, arXiv:math/1008.0973v1.
3. D. McDuff and D. Salamon, *Introduction to symplectic topology*.
4. I. Vaisman, *Lectures on the geometry of Poisson manifolds*.
5. K. H. Bhaskara and K. Viswanath, *Poisson algebras and Poisson manifolds*.
6. D. Huybrechts, *Complex Geometry: An Introduction*.
7. S. Kobayashi, *Differential Geometry of Complex Vector Bundles*.
8. F. Kirwan, *Cohomology of quotients in symplectic and algebraic geometry*.
9. J. M. Lee, *Introduction to smooth manifolds*.

Method of evaluation: Your final grade will be based on **five assignments**, to be handed in class about every two weeks.

Note: Assignments will be posted on the course webpage.

Academic Integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. [Check http://www.uwaterloo.ca/academicintegrity/for more information.]
Grievance: A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4, http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm. When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity to avoid committing academic offenses and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course professor, academic advisor, or the undergraduate associate dean. For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline, http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm. For typical penalties check Guidelines for the Assessment of Penalties, http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm.

Appeals: A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72, Student Appeals, http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm.

Note for students with disabilities: The Office for Persons with Disabilities (OPD), located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the OPD at the beginning of each academic term.