

PMATH 955 -- Topics in Geometry
Winter 2010
Course Outline

Instructor: R. Moraru

Course title: Topics in Geometry: Gauge Theory

Lectures: MW 9:00-10:20, MC5046

Office hours (MC 5170): W 11:00 – 12:00 and Th 15:00-17:00 (subject to change) or by appointment.

Course web page: <http://www.math.uwaterloo.ca/~moraru/955.html>

Overview: This course will be an introduction to gauge theory. It will cover material that any graduate student interested in geometry will find useful (such as bundles, connections, characteristic classes, Hodge Theory, and Yang-Mills Theory). The course should be accessible to students who have taken PMATH 465 (Differential Geometry) or an equivalent course. For students who do not have the required background, it is highly recommend that they also attend the lectures of PMATH 465, which is taught on Tuesdays and Thursdays in Winter 2010.

Topic to be covered:

(a) *Basics:*

- Review of manifolds, vector fields, differential forms.
- Vector bundles and principal bundles: definitions and basic constructions.
- Connections, curvature, and gauge groups.
- Covariant derivatives and holonomy.
- Characteristic classes and Chern Weil Theory.

(b) *More specialized topics:*

- Flat vector bundles and flat connections.
- Metric connections on vector bundles.
- Some important equations of gauge theory: Yang-Mills, anti-self-duality, Hermitian-Einstein.
- Instanton moduli spaces.

(c) *Some important theorems and applications:*

- Flat connections and representations of the fundamental group.
- Stable holomorphic bundles and the Kobayashi-Hitchin correspondence.
- Donaldson and Seiberg-Witten invariants (time permitting).

(d) *Potential additional topics:*

- Berger's classification of Riemannian holonomy groups.
 - Generalisations to higher dimensional manifolds of instantons on four-manifolds.
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Some useful references:

- “Differential Geometry of Complex Vector Bundles”, S. Kobayashi, Princeton, 1987;

- “Complex Geometry: An Introduction”, D. Huybrechts, Universitext, Springer, 2004;
 - “Characteristic classes”, J. Milnor and J. Stasheff, Princeton, 1974;
 - “An introduction to gauge theory”, J. W. Morgan, in: Gauge theory and the topology of four-manifolds, ed. R. Friedman and J. W. Morgan, IAS/Park City Mathematics Institute, 1994;
 - “The Geometry of Four-Manifolds”, S. K. Donaldson and P. B. Kronheimer, Oxford Mathematical Monographs, Oxford University Press, 1997.
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Method of evaluation: Your final grade will be based on 3 assignments, to be handed in class every three weeks, one take-home final examination, one talk, and one 10--15 page paper on a topic related to the course.

Grading scheme: Assignments 60%, Take-home final exam 10%, Talk 10%, Paper 20%.

Tentative schedule of due dates (subject to change):

- Friday, 29 Jan.: Assignment 1.
- Friday, 26 Feb.: Assignment 2.
- Friday, 19 Mar.: Assignment 3.
- *Monday, 5 Apr.: Take-home final exam Midterm (due by noon on Wednesday, 7 Apr.).*
- Monday and Tuesday, 19 and 20 Apr.: Talks.
- Friday, 23 Apr.: Papers are due.