# PMATH 764: Assignment 3

Due: Friday, 14 June, 2013.

1. Determine whether or not the following are varieties.
   (a) The orthogonal group $O(n, k)$, where $k$ is an algebraically closed field with $\mathrm{char}(k) \neq 2$.
   (b) The special unitary group $SU(2, \mathbb{C})$.
   (c) $V(xz - y^2, yz - x^3, z^2 - x^2y) \subset \mathbb{C}^3$.
2. Let $X$ and $Y$ be two varieties and $\phi : X \to Y$ be a polynomial map.
   (a) Show that $\phi^*$ is injective if and only if $\overline{\phi(X)} = Y$.
   (b) Show that $\phi^*$ is surjective if and only if $\phi$ has a polynomial left inverse.
3. Let $k$ be an algebraically closed field with characteristic $p > 0$. Consider the map $\phi : \mathbb{A}^1 \to \mathbb{A}^1$ defined by $t \mapsto t^p$; this is called the *Frobenius morphism*. Show that $\phi$ is bijective but not an isomorphism.
4. Show that $X = V(y^2 - x^3) \subset \mathbb{A}^2$ is not isomorphic to $\mathbb{A}^1$.

   *Note:* You may assume that the field over which you are working is infinite.