PMATH 465/665: Assignment 1

Due: Monday, 26 September, 2011

Suggested problems:

1. Problem 1-2, p. 28.
2. Let X be a Hausdorff, second countable topological space, and let $Y \subset X$ be a subset. Show that the relative topology on Y is also Hausdorff and second countable.
3. Problem 1-4, p. 28.
4. Problem 1-6, p. 29.

Problems to be handed in:

1. Problem 1-5, p. 28, parts (a), (b), (c).
2. *Real projective n-space* $\mathbb{RP}^n$.
   (a) Let S^n be the unit sphere in $\mathbb{R}^{n+1}$, and consider the quotient space $S^n/\sim$, where $x \sim y$ if and only if $x = \pm y$. Show that $S^n/\sim = \mathbb{RP}^n$.
   (b) Prove that $\mathbb{RP}^n$ is compact and connected.
3. Problem 1-7, p. 30.
   Note: For compactness, show that $\mathbb{CP}^n = S^{2n+1}/\sim$, where $x \sim y$ if and only if $x = ty$ for some $t \in \{a \in \mathbb{C}^* : |a| = \sqrt{a\bar{a}} = 1\} = S^1$.
4. (a) Let M and N be C^k manifolds. Use the manifold construction lemma to show that their (Cartesian) product

$$M \times N = \{(x, y) : x \in M \text{ and } y \in N\}$$

   is also a C^k manifold. Deduce that finite products $M_1 \times \cdots \times M_r$ of C^k manifolds are C^k manifolds.
   (b) A *real n-torus* $S^1 \times \cdots \times S^1$ is defined as the product of n copies of S^1. Prove that n-tori are smooth compact, connected, n-dimensional manifolds.