CO778 Portfolio Optimization
Department of Combinatorics and Optimization
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PORTFOLIO OPTIMIZATION — Feedback on Assignment 1

I. General feedback

1. Claims without justification receive no marks.
2. When MATLAB code and/or output is required in the question, a solution without the printed code is considered in complete.
3. Performance of roughly between one-half and one-third of the class on Ex.1.9 and (especially) 1.11 is not good. If you had trouble doing these questions, make sure you read the solution posted online.
4. *Linear algebra*
   (a) (**Ex. 1.7**) It is impossible to take the inverse of a n-vector (for $n > 1$), nor that of a rectangular matrix.
   Even if the matrix is a square one, you have to check if it is invertible before taking inverse.
   (b) When you do matrix multiplication, please pay attention the the dimension: for two matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$, the matrix product AB makes sense only if $n = p$.
   (c) (**Ex. 1.7**) If $C \in \mathbb{R}^{n\times n}$ and $x \in \mathbb{R}^n$ ($n > 1$), Cx^T does not make sense due to dimension mismatch. In particular,
   $$C(x'Cx) \neq (Cx') = Cx.$$
   (But it is possible to do the following:
   $$x'\Sigma^{-1}x = x'I\Sigma^{-1}x = x'\Sigma^{-1}\Sigma\Sigma^{-1}x = (\Sigma^{-1}x)'\Sigma(\Sigma^{-1}x).\quad)$$
5. *Multivariate calculus*
   (a) *Partial derivatives.* For a function $f : \mathbb{R}^n \to \mathbb{R}$ and $x = (x_1, \ldots, c_n) \in \mathbb{R}^n$, we can talk about
   $$\frac{\partial f}{\partial x_i} \text{ , but not } \frac{\partial f}{\partial x},$$
   (that is, you cannot take partial derivative with respect to a vector.)

II. Question-specific feedback

Ex. 1.6. The main points of the exercise is for you to do the differentiation, and then to set it to zero to get the optimality conditions (which is the easy part). Many people write down something like $\partial f/\partial u$ in their solution; such an answer is considered to be incorrect. (See I.2(a) above.)

Ex. 1.7. A symmetric matrix is non-singular if and only if *all* eigenvalues are non-zero (not just one). Also see I.1 above.

Ex. 1.8. Most people forgot to point out that $C = HH'$ is symmetric.

Ex. 1.9. When you say x is an optimal solution, you need to state the optimization problem you are looking at.

Ex. 1.9. Some people consider the optimization problem

$$\min \frac{1}{2}s'Cs \quad \text{s.t.} \quad 0s = 0,$$

so that they can apply Theorem 1.4. This is, though mathematically valid, unnecessary, as we may apply on the unconstrained problem $\min s'Cs$ the following simpler result that is typically covered in basic calculus class:

If $f : \mathbb{R}^n \to \mathbb{R}$ attains a local minimum at $\bar{x} \in \mathbb{R}^n$ and if f is differentiable, then $\nabla f(\bar{x}) = 0$.

Ex. 1.11(a). Explicit solutions given without intermediate steps of computation is considered to be (very) incomplete.

Ex. 1.11(a). Note that $\|x - d\|^2 = \frac{1}{2}x'(2I)x - (2d)'x + \|d\|$, so in the MATLAB code you need to put $c = 2d$.

Ex. 1.11(a). Some people gave the proof as follows:
first note that if we let $f(x) = x'Cx - c'x$, where $C = 2I$ and $c = -2d$, and $A = [a_1, \dots, a_n]$ (note that A is a row vector), then the problem is equivalent to $\min\{\frac{1}{2}x'Cx + c'x : Ax = b\}$. Suppose $x \in \mathbb{R}^n$ is an optimal solution. Then there exists a multiplier $u \in \mathbb{R}$ s.t.

$$\begin{aligned}
\begin{cases} -\nabla f(x) &= A'u \\ Ax &= b \end{cases} &\Rightarrow \begin{cases} -Cx - c &= A'u \\ Ax &= b \end{cases} \\
&\Rightarrow \begin{cases} x &= -(C^{-1})'A'u + (C^{-1})'c = -(C^{-1})'A'u - d \\ Ax &= b \end{cases} \\
&\Rightarrow b = Ax = -A(C^{-1})'A'u + Ad = -2\|a\|^2 u + a'd \\
&\Rightarrow u = \frac{a'd - b}{2\|a\|^2}.
\end{aligned}$$

The only thing is, no one actually simplified the expression $A(C^{-1})'A'$, which is okay, but at least one such point out that $A(C^{-1})'A'$ is indeed a *positive* scalar, by the fact that A is indeed a row vector and the positive semi-definiteness of C^{-2}, so we can divide both sides of $b = -A(C^{-1})'A'u + Ad$ by $A(C^{-1})'A'$.

Prepared on February 5, 2010.